Properties of stabilizer Rényi entropy and entanglement distributions in random circuits

Gergely Zaránd
D. Szombathy, A. Valli, C. P. Moca, L. Farkas,
J. Asbóth, T. Rakovszki

Budapest University of Technology and Economics, Műegyetem rkp. 3, 1111, Budapest, Hungary

zarand.gergely.attila@ttk.bme.hu

Although it may still be somewhat elusive, quantum complexity is a central theme in quantum computing. Within the framework of quantum resource theory, entanglement and nonstabilizerness (or "magic") play a prominent role in its characterization.
Entanglement is often regarded as the main discriminant for quantumness but it is not exhaustive, as highly-entangled states that can be produced by Clifford circuits are efficiently simulable classically. The lack of complexity of these states is encoded in the peculiar spectral properties characteristic of Clifford circuits [1].
On the other hand, magic — as measured by the stabilizer Rényi entropy, quantifies the non-Clifford resources required to prepare a quantum state and is pivotal for quantum advantage. We obtain both the magic and entanglement probability distributions by a numerically sampling of Haar-random unitary circuits. We find that both are found highly concentrated around the typical values, with a variance that is exponentially suppressed with system size (N). Typical Haar states are resourceful — i.e., characterized by magic and entanglement of $\mathcal{O}(N)$ [2].
Furthermore, while these two resources are related (e.g., entanglement is instrumental to generating high-magic states) little is known about their relationship. Considering the joint distribution, we find that the magic and entanglement fluctuations surprisingly are asymptotically independent, as both the covariance and the mutual information of the joint distribution vanish exponentially in the thermodynamic limit [2].

References

[1] D. Szombathy, et al., Phys. Rev. Research **7**, 043080 (2025)
[2] D. Szombathy, et al., Phys. Rev. Research **7**, 043072 (2025)

Figures

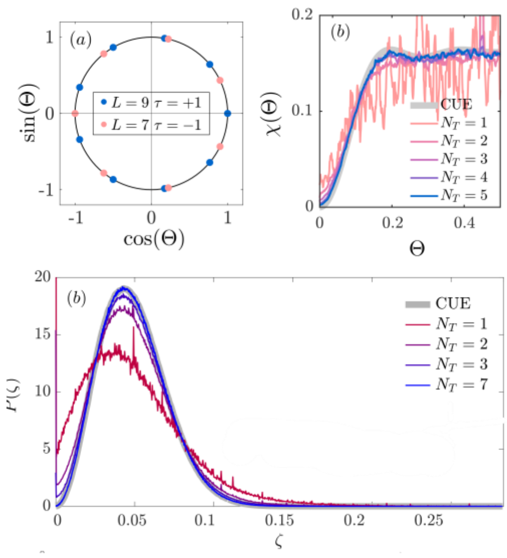

Figure 1: (a) eigenvalues of Clifford operators; (b) phase correlation function and (c) level spacing statistics convergence to the CUE result as a function of T-gate doping Clifford circuits.

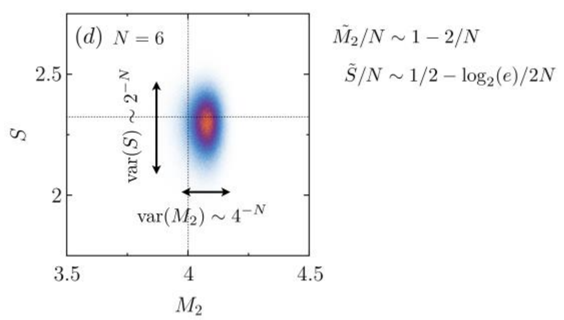

Figure 2: Concentration of the joint magic and entanglement probability distribution around the typical values.