Quantum Fisher Information and the Curvature of Entanglement

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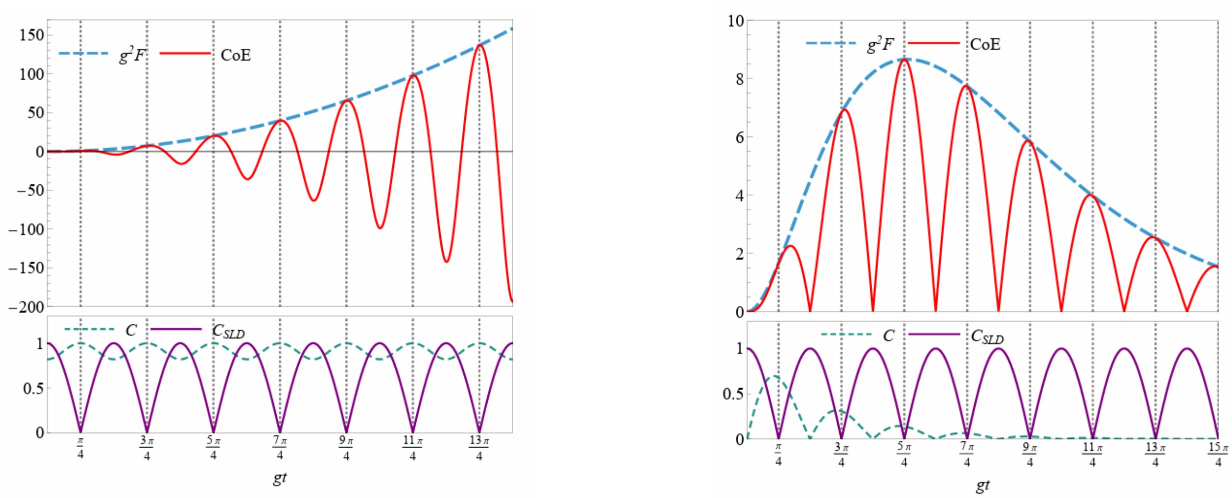
Abstract

We analyze a time-resolved connection between metrological sensitivity and entanglement in a two-qubit quantum probe used to estimate an interaction coupling parameter g. Sensitivity is quantified by the quantum Fisher information (QFI), while entanglement is quantified by concurrence C. To place both quantities on equal footing with respect to the *estimated* parameter, we introduce the curvature of entanglement $\mathrm{CoE} \equiv -\partial_g^2 C$, which characterizes the local sharpness of entanglement as a function of g and is positive at concurrence maxima. For unitary pure-state dynamics generated by a generic two-qubit interaction Hamiltonian—reducible by local rotations to an anisotropic Heisenberg form—we show for broad families of initial states that CoE is upper-bounded by the QFI, and that the bound is saturated at special instants corresponding to maximal entanglement. We further analyze the symmetric logarithmic derivative (SLD) associated with estimation of g and demonstrate that, precisely at the saturation points $\mathrm{CoE} = \mathrm{QFI}$, the SLD eigenstates become separable for physically relevant state families. As a result, optimal parameter estimation can be achieved using local product measurements, whereas away from these instants entangled measurements are generically required to attain the quantum Cramér–Rao bound. We show that this correspondence remains robust under Markovian amplitude damping described by a Gorini–Kossakowski–Sudarshan–Lindblad master equation: although concurrence decays in time, the coincidence between concurrence maxima, $\mathrm{CoE} = \mathrm{QFI}$, and separability of the SLD eigenbasis persists. These results identify experimentally accessible operating points at which entanglement-enhanced sensing is simultaneously near-optimal and measurement-friendly.



(a) Closed system

(b) Open system

Figure 1: Representative dynamics for closed and open systems showing that the curvature of entanglement touches the quantum Fisher information at concurrence maxima, coinciding with vanishing entanglement of the SLD eigenstates.

Reference

[1] Z. H. Saleem *et al.*, *Quantum Fisher Information and the Curvature of Entanglement*, arXiv:2504.13729v2 (2026).