Small-world Entanglement Landscapes Yield *O(N Log N)* QKD Scaling

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Abstract

Entanglement-based QKD protocols are often thought to face a fundamental N(N-1)/2 authentication bottleneck, arising from the presumed need to establish pairwise trust between all node pairs in a network of N users. Here we show that this worst-case scaling is physically unattainable in realistic quantum networks. Lossy channels and measurement back-action induce finite E91 correlation lengths, effectively screening the set of node pairs that can violate a CHSH threshold. We model entanglement distribution under realistic operations, including Clifford processing, Bell measurements, and feed-forward by treating the network as an open quantum system on a graph and tracking Bell correlators through Pauli transfer-matrix methods. The resulting entanglement heat maps reveal a sparse, small-world structure of high-fidelity E91-usable links satisfying N ≤ |E| ≤ O(N logN, where the logarithmic factor reflects the typical path lengths of small-world infrastructures. Consequently, the authenticated degree scales as O (N logN), not as a protocol assumption but as an emergent property of entanglement propagation under loss. Authentication complexity is therefore governed by physical correlation structure rather than by combinatorial worst-case arguments, resolving the apparent quadratic barrier to scalable entanglement-based QKD.

References

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Figures

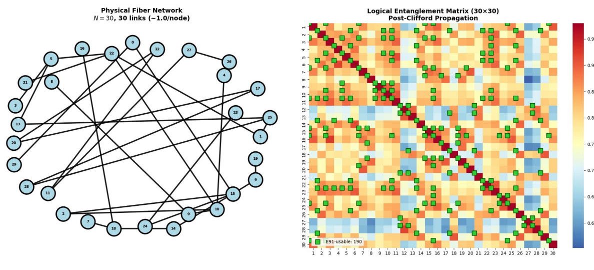

Figure 1: Entanglement distribution of EPR pairs in a small-world network simulation.
Left: Physical N=30, E=24 links (the average degree being ⟨k⟩=2E/N=1.6).
Right: 30×30 probability configuration S≃14 usable pairs. (S~0.5N), as predicted. The authentication complexity being then |E_E91|=80~O(N ln N)=102.