Quantum Metrology with Many-Body Systems

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Quantum metrology seeks to achieve the most precise measurements permitted by the laws of quantum mechanics [1]. In the first part of this Tutorial, we will introduce the theoretical foundations of the field, starting from the Cramér-Rao bound and the Quantum Fisher Information (QFI). For an N-body sensor estimating an unknown parameter θ, we will demonstrate how entangled states enable a transition from the classical Shot-Noise Limit ($\Delta\theta \geq 1/\sqrt{N}$) to the Heisenberg Limit ($\Delta\theta \geq 1/N$). However, we will argue that such a quantum advantage is fragile and typically lost in the presence of dissipation or decoherence.

This challenge motivates the second part of the tutorial: Many-Body Metrology (see Fig. 1). Here, the presence of interactions between particles enables new strategies for high-precision sensing that remain robust even in the presence of noise, providing a new avenue for enhanced quantum metrology. In particular, we will explore how collective many-body phenomena, such as dissipative and quantum phase transitions, enable enhanced sensitivity in the steady state. We will provide a brief overview of this rapidly growing field [2].

Finally, we will explore the fundamental precision limits of many-body sensors, focusing on recent results concerning the ultimate bounds of precision [3, 4]. We will derive upper bounds for the QFI of ground and thermal states, and contrast them with the dynamical Heisenberg limit (see Table 1). If time allows, we will provide examples of many-body systems—such as interacting spin chains—approaching these theoretical limits.

References

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Figures

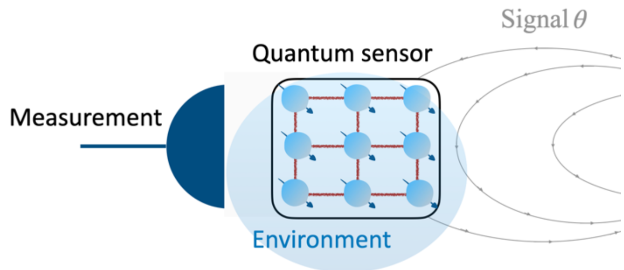

Figure 1: Sketch of a many-spin sensor.

Dynamical	Gibbs ensemble	Gapped ground state
$e^{-iH_\theta t}\rho e^{iH_\theta t}$	$\frac{e^{-\beta H_\theta}}{Z}$	$\lim_{\beta\to\infty} \frac{e^{-\beta H_\theta}}{Z}$
$\mathcal{F}_\theta \leq t^2 N^2$	$\mathcal{F}_\theta \leq \frac{\beta^2}{4} N^2$	$\mathcal{F}_\theta \leq \frac{N^2}{\Delta^2}$

Figure 2: Upper bounds on the QFI, F_θ, for different encodings, based on [3].