

Berryology and Beyond: From Quantum Geometry to Topological Order

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Geometric properties of quantum states—encoded in the **Berry curvature**, **Berry phase**, and the **quantum metric**—provide a unifying language connecting phenomena across transport, thermodynamics, and optical response. In the first part of this talk, I will introduce these foundational concepts and highlight how band geometry manifests in measurable observables ranging from anomalous Hall transport to orbital magnetization, superfluid stiffness, and light-matter coupling. Building on this framework, I will then discuss how **interactions** reshape quantum geometry, enabling phases that have no non-interacting analogs. Particular emphasis will be placed on **fractional Chern insulators** and the interplay of **symmetry breaking** and **topological order** in flat-band settings.

The second part presents a series of **case studies** drawn from contemporary platforms: graphene-based moiré materials, engineered lattice systems, and unconventional superconductors. I will summarize what aspects of their emergent correlated and topological behaviors are on firm theoretical and experimental footing, which interpretations remain under debate, and where decisive future progress—both experimental and conceptual—may arise. Throughout, the aim is to illustrate how quantum geometry provides a powerful organizing principle for understanding and designing quantum materials with exotic collective properties.

Figures

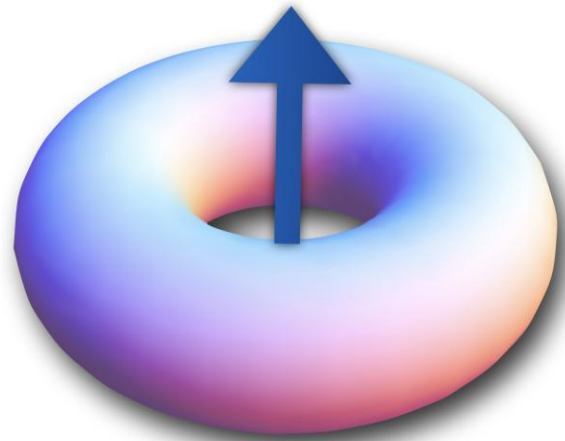


Figure 1: Topological states are identified and distinguished through global properties of the wave function on a control space, e.g. organized by fluxes or momenta. Quantum geometry, in contrast, includes both local and global properties on this space, and is thus relevant for energetic properties as well universal topological ones.
