Luttinger's Theorem Violation and Green's Function Topological Invariants in a Fractional Chern Insulator

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Luttinger's theorem constrains the particle density of interacting fermions through global properties of the single-particle Green's function, and its violation signals a breakdown of the identification between the quantized Hall response and the Green-function-based Ishikawa-Matsuyama invariant. This phenomenon becomes especially compelling in strongly correlated topological phases, such as fractional Chern insulators, where fractionalized quasiparticles lack an adiabatic connection to electrons, raising the question of how Green's-function-based topological invariants manifest in such phases. Using exact diagonalization of the fermionic Harper-Hofstadter-Hubbard model, we compute bulk single-particle Green's functions deep inside a fractional Chern insulating phase and directly evaluate the Luttinger count, its possible correction (the Luttinger integral), and the Ishikawa-Matsuyama invariant $N_3[G]$. We demonstrate a clear violation of Luttinger's theorem and show that the fractional nature of the many-body Chern number is encoded in the Streda response of the Luttinger integral, while the integer invariant $N_3[G]$ arises from the Streda response of the Luttinger count. We also analytically prove that $N_3[G]$ is fully determined by the Luttinger count together with the Chern number of the occupied Bloch band, upon neglecting Bloch-band mixing. Finally, we propose an experimental protocol to extract all Green-function-based topological invariants from local density-of-states measurements, experimentally accessible in fractional quantum Hall systems.

References

[1] Markov, A. A., Nikishin, A. M., Cooper, N. R., Goldman, N., & Gavensky, L. P. (2026). arXiv preprint arXiv:2603.17006.

Figures

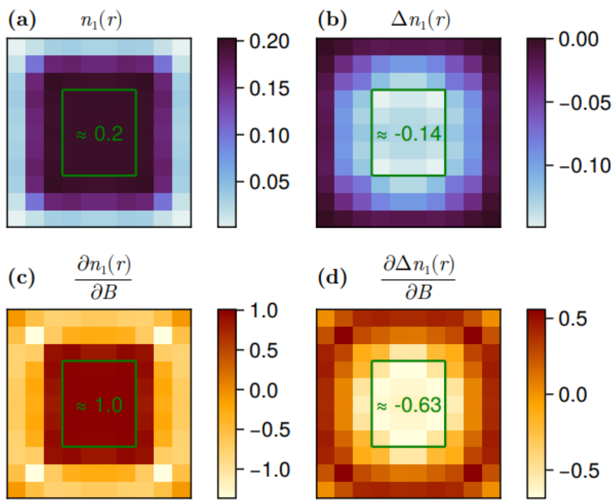

Figure 1: Spatial distributions of the local Luttinger count $n_1(\mathbf{r})$ and Luttinger integral $\Delta n_1(\mathbf{r})$. The presence of a finite, negative $\Delta n_1(\mathbf{r})$ in the bulk directly signals the breakdown of Luttinger's theorem. Streda responses of the Luttinger count and the Luttinger integral densities. The derivative of $n_1(\mathbf{r})$ with respect to magnetic field yields an integer-valued response, corresponding to an Ishikawa-Matsuyama invariant $N_3[G]$ ≈1$. By contrast, the Streda response of $\Delta n_1(\mathbf{r})$ is fractional and accounts for the difference between $N_3[G]$ and the many-body Chern number.