Exploring Variational Entanglement Hamiltonians

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Recent advances in analog and digital quantum-simulation platforms have enabled exploration of the spectrum of entanglement Hamiltonians via variational algorithms [1]. The entanglement Hamiltonian H_A parameterizes the reduced density matrix by rho_A=exp(-H_A) and its spectrum, the entanglement spectrum, has become an important diagnostic for topologically ordered phases and symmetry-protected topological phases [2,3] since the seminal work by Li and Haldane [4]. In our contribution we analyze the convergence properties of the variationally obtained solutions and compare them to numerically exact calculations in quantum critical systems. We demonstrate that interpreting the cost functional as an integral permits the deployment of iterative quadrature schemes, thereby reducing the required number of measurements by more than an order of magnitude even in the presence of noise. We further show that a modified ansatz captures deviations from the Bisognano–Wichmann (BW) form in lattice models, improves convergence, provides a cost-function-level diagnostic for quantum phase transitions and improves trainability. Finally, we establish that a low cost value does not by itself guarantee convergence in trace distance. Nevertheless, it faithfully reproduces degeneracies and spectral gaps, which are essential for applications to topological phases [5].

References

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[4] H. Li, F. D. M. Haldane, Phys. Rev. Lett. **101,** 010504 (2008)
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Figures

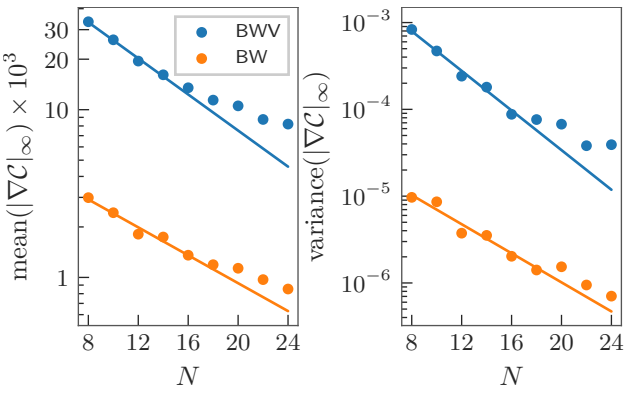

Figure 1: Mean (left) and variance (right) of the gradient for the BW-like (BW) and BW-violating (BWV) ansatz vs. system size N. The flattening is indicative of subexponential scaling with system size for the BWV ansatz.

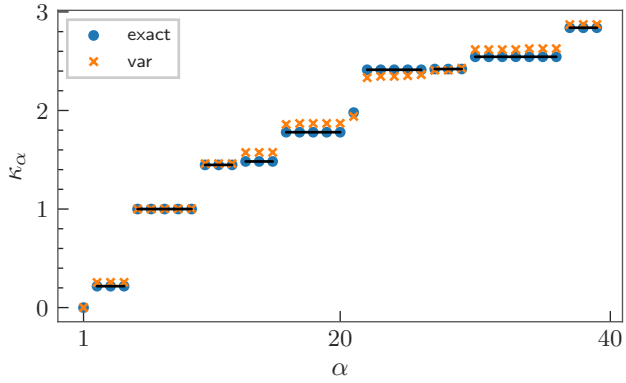

Figure 2: Exact and variational (var) entanglement spectrum in a topological Haldane phase with the BWV ansatz, which resolves degeneracies with high resolution.