Kramers-Kronig detection in the quantum regime

Nicolas Fabre
Thomas Pousset, Maxime Federico, Romain Alléaume

Telecom Paris, Institut Polytechnique de Paris, 19 Place Marguerite Perey, 91120 Palaiseau, France
nicolas.fabre@telecom-paris.fr

Abstract

We investigate the quantum formulation of the Kramers–Kronig detection (KKD) technique, originally developed for classical optical communications (See Fig.1) [1,2]. KKD consists in mixing an unknown optical field with a strong monochromatic local oscillator (LO) on an unbalanced beamsplitter, followed by direct intensity detection on a single photodiode. When the signal satisfies the minimal phase and single sideband (SSB) constraints, the phase of the optical field can be reconstructed from the measured intensity via a Hilbert transform, allowing retrieval of both field quadratures.

We demonstrate that this reconstruction remains valid in the quantum regime to first order in the LO amplitude, showing that KKD acts as a coherent, Gaussian measurement operationally equivalent to double homodyne detection (DHD), albeit with a 3 dB signal-to-noise ratio (SNR) loss due to coupling with an orthogonal spectral mode. Using a normalized spectral mode formalism, we analyze the role of the spectral degree of freedom, crucial in both classical and quantum KKD, and clarify the definition of the Hermitian phase operator associated with intensity measurements. This operator, distinct from standard quantum-optical phase operators, retrieves the relative phase between the LO and the quantum state, encoding both particle-number statistics and temporal mode structure.

Finally, we propose a spectral tomography protocol for single-photon states inspired by KKD. Relying on spectral engineering, temporal-resolved detection, and KK relations, the method enables full reconstruction of the spectral-temporal wavefunction of single photons under the minimal phase and SSB constraints.

Furthermore, by employing a direct detection approach, KKD replaces the optically demanding configuration of traditional balanced detection with digitally implemented signal reconstruction and processing This transition effectively eliminates susceptibility to technical noise sources associated with finite common-mode rejection ratios, which can otherwise degrade the performance of quadrature measurements in balanced coherent receivers. In addition, by eliminating the need for balanced detection schemes, KK detection can significantly reduce the number of required detectors in quantum networks, thereby simplifying receiver design and improving scalability without compromising measurement sensitivity.

References

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[3] Liu, X. *et al.* Robust and cost-effective quantum network using Kramers-Kronig receiver. https://doi.org/10.48550/arXiv.2509.06711 (2025).

Figures

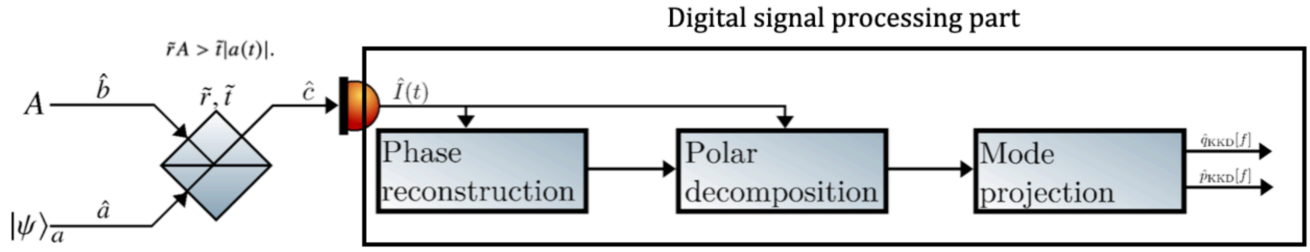

Figure 1: Setup for Kramers-Kronig detection. For Kramers-Kronig detection (a), the signal of interest is injected in the a input of a highly unbalanced beamsplitter while a monochromatic strong and classical local oscillator is injected in the b input. The signal of interest verifies the single sideband constraint. A single output intensity is measured by a photodiode and the intensity at each instant is recorded. The phase of the output c is reconstructed by taking the Hilbert transform of the logarithm of the measured intensity.