

Unavoidable entropy production in fault-tolerant quantum error correction

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The aim of quantum error correction (QEC) is to counteract the decoherence and relaxation of qubits, which arises from their unavoidable interaction with the environment. An important and largely unexplored question is the fundamental energetic cost of such process.

In this work, we investigate the thermodynamic limits of fault-tolerant quantum error-correcting circuits, focusing on the minimum amount of entropy that is unavoidably created by fault-tolerant QEC.

Previous studies [1] have shown that, when neglecting the noise during the correction process, QEC can in principle be performed in a thermodynamically reversible way. In this work, by contrast, we explicitly account for faults occurring during the error-correction operations themselves (fault tolerant QEC). We show that when decoherence and relaxation affect the circuit during the error-correcting operations, QEC can no longer be thermodynamically reversible. Unwanted correlations generated during the process must be dissipated at each cycle, leading to an unavoidable finite entropy production.

We derive a lower bound on the entropy generated by fault-tolerant QEC of order $p \ln[p]$ per logical qubit and per correction cycle, where p is the probability of a dissipation-induced error per gate operation. We argue that this lower-bound is generic and consists of two contributions: one arising from the nonequilibrium nature of the qubits, common to both ideal and

fault-tolerant QEC ; and a novel contribution specific to fault tolerance, originating from the propagation of errors from ancillary qubits to data qubits.

Our results are obtained from the generalisation of a detailed analysis of entropy flows in fault-tolerant three-qubit error correction and are supported by an analogy with Maxwell's demon. In this perspective, the ancillary qubits act as a noisy memory whose reset, imposes an unavoidable thermodynamic cost. Fault-tolerant QEC can therefore be viewed as an imperfect Maxwell's demon, closely analogous to scenarios involving imperfect measurements [2].

References

- [1] Nielsen, Caves, Schumacher, Barnum, Proc. A , 1998.
- [2] A Danageozian, MM Wilde, F Buscemi - PRX Quantum, 2022