Hybrid Krylov-Subspace Methods for Solving Non-Linear PDEs on Quantum Computers

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Abstract

Numerical solvers for Partial Differential Equations (PDEs) are of great interest in various domains, e.g., in aerodynamics or for transport equations in electrochemistry [1], and the need for fine-grained solutions of non-linear PDEs is growing. While subspace methods allow for a dimensionality reduction, non-linear PDEs require linearization schemes, such as the Carleman-linearization [2], resulting in linear systems of exponential dimensionality, operating on the limits of classical methods. Motivated by Krylov-subspace methods [3], which find approximate solutions in iteratively growing subspaces, the aim of this talk is to investigate the potential of two existing methods from quantum computing to compose a NISQ-era hybrid quantum-classical algorithm. Firstly, non-linear quantum computing (QNPUs) [4,5] and secondly Quantum Subspace Expansion (QSE) [6], both promising tools towards more scalable computations. The use of QNPUs enables linearization in a tensor-product-subspace using ancilla qubits, while offering efficient gate-based implementations. On the other hand, QSE measures high-dimensional overlaps on a quantum computer. The combination of these two methods yields the possibility of encapsulating the high-dimensional steps of linearization and subspace projection on a quantum device. As a result, only a lower-dimensional subspace problem remains to be solved on classical hardware.

References

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