ZX-diagrams as a Graphical Diagnose of Topological Order

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Abstract

The topological entanglement entropy γ [1,2] of a system's bipartition (A,B), as in Fig.1, is a key diagnostic of the type of topological order, as it measures long-range entanglement. However, γ is not universal [3,4]. For example, cluster states, which should be trivial (γ = 0), can have γ > 0 [5]. In addition, γ can be computationally expensive to obtain. In this work, we introduce a new characterization of topological order, which helps to circumvent practical and non-universal issues in computing γ. The boundary-reduced density matrix $\rho_{\partial A}$ is a reduced density matrix that keeps the relevant degrees of freedom at the boundary of the bipartition. When simplified as a ZX diagram, as we show in Fig.2, it has an invariant number of nonlocal nodes which matches with γ. We have observed consistent results, which we show in Fig.2, for the toric code (γ = 1), the hexagonal toric code (γ = 1), the color code (γ = 2) and cluster states with γ > 0.

References

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Figures

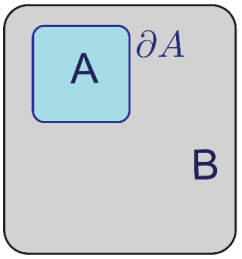

Figure 1: Bipartition of a system into two regions A and B with a boundary ∂A.

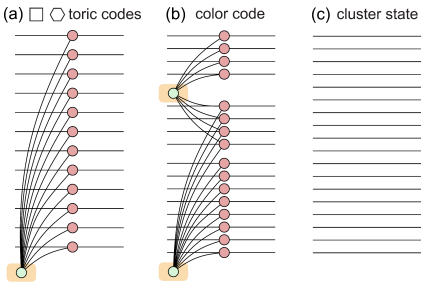

Figure 2: The boundary-reduced density matrix $\rho_{\partial A}$ as ZX diagrams for (a) toric codes, (b) color code, and (c) cluster state. The horizontal lines are the boundary degrees of freedom. The green nodes connect nonlocally to the red nodes, a signature of long-range entanglement, and its number is an invariant that characterizes the type of topological order, since it matches with γ.