Length-scale sensitivity of quantum mutual information variants

Patrick M. Lenggenhager[1]
Mark Fisher[2], Titus Neupert[2]

[1] *Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, Dresden, Germany*
[2] *University of Zurich, Winterthurerstrasse 190, 8057 Zurich, Switzerland*

plengg@pks.mpg.de

Information-theoretic measures of correlations have proven to be powerful tools in characterizing and classifying quantum many-body systems, providing insights into ground-state properties, critical phenomena, and non-equilibrium dynamics. Among these, mutual information (MI) serves as a versatile quantity, capturing both classical and quantum correlations. However, there exist multiple inequivalent definitions of MI in the quantum setting, differing in both their information-theoretic properties and computational complexity. This raises the question of how these different variants behave in practical settings, particularly in numerical simulations, where discretization, finite-size effects [1], and approximations play a crucial role.

In this talk, I will present a systematic study of how different Rényi MI variants behave in lattice models, particularly in their ability to resolve long-wavelength properties. Using ground states of critical one-dimensional systems as a testbed, we reveal that MI variants of the so-called α-z-family [2,3] fall into three distinct regimes, illustrated in Figure 1: (i) those that access continuum physics at all scales, (ii) those requiring a finite minimal subsystem size, and (iii) those dominated by short-wavelength effects, making the continuum limit inaccessible. To quantify these differences, we introduce a characteristic length scale ℓ^* for each MI variant, which dictates the minimal subsystem size required to recover long-wavelength properties.

These insights provide practical guidance for selecting MI variants in numerical studies and highlight the interplay between information-theoretic definitions and emergent physical properties in lattice systems. As a proof of principle, we demonstrate this by implementing different Rényi MI variants using the matrix product operator framework and applying it to the transverse-field Ising model.

References

[1] V. Alba et al., Phys. Rev. B 81, 060411(R) (2010)
[2] K. M. R. Audenaert et al., J. Math. Phys. 56, 022202 (2015)
[3] J. Kudler-Flam et al., J. High Energy Phys. 06, 195 (2024)

Figures

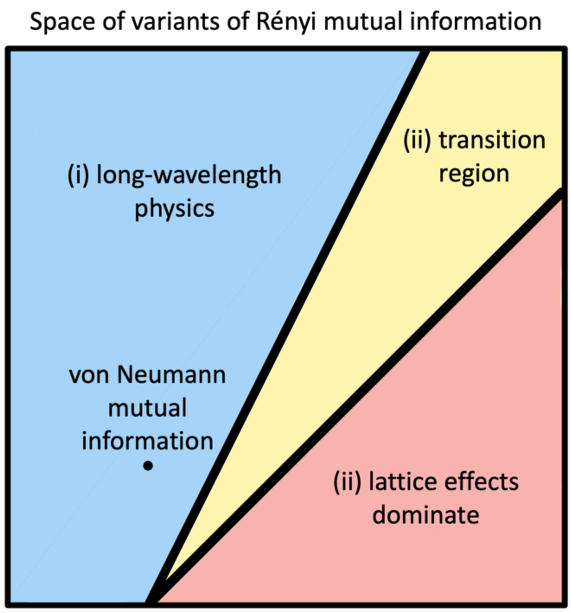

Figure 1: Illustration of regions of different sensitivity to long- vs short-wavelength physics, i.e., lattice effects, in the space of Rényi mutual information variants. More specifically, the subspace parametrized by the α-z-Rényi mutual information is shown. The von Neumann mutual information is marked by a black dot.