Penrose Spin Calculus: ZX for SU(2)

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We introduce the Penrose spin calculus as an elevation of Penrose's diagrams and associated Binor calculus [1] to the level of a formal diagrammatic language. By leveraging the mixed-dimensional ZX calculus[2], a complete language for finite dimensional Hilbert spaces, we formulate a diagrammatic language for SU (2) representation theory in quantum informational terms. Using this language we firstly articulate the classic angular moment relations diagrammatically. Next, we present the symmetric projectors required to transition between tensor products of qubits to spin-j. We then demonstrate their effectiveness by deriving the classic spin coupling objects the 3jm Wigner symbols along with all their core relationships again diagrammatically [3]. The versatility of the Penrose spin calculus is demonstrated through applications in permutational quantum computing, condensed matter physics, quantum machine learning, and quantum gravity. Offering the analysis of permutational computing transition amplitudes[4], analysis of AKLT states[5], the evaluation of barren plateaus for SU (2) symmetric ansatze[6], and the derivation of the minimum quantised volume in loop quantum gravity respectively[7]. In each domain, we derive our results purely through diagrammatic reasoning some of which are entirely new and some of which are more natural simplifications of previous diagrammatic techniques making use of the Penrose calculus' natural presentation of SU(2) representations. Our results establish the Penrose spin calculus as a powerful tool for representing and computing with SU(2) systems graphically, offering new insights into foundational relationships and paving the way for new diagrammatic algorithms for theoretical physics.

References

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Figures

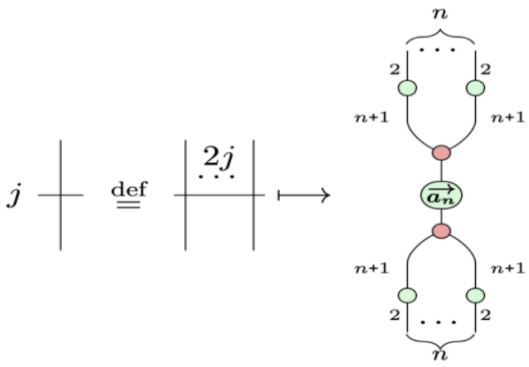

Figure 1: On the right the a spin-J degree of freedom decomposed into Penrose's Binor diagrams in the centre, mapped to the ZX Penrose calculus on the right.

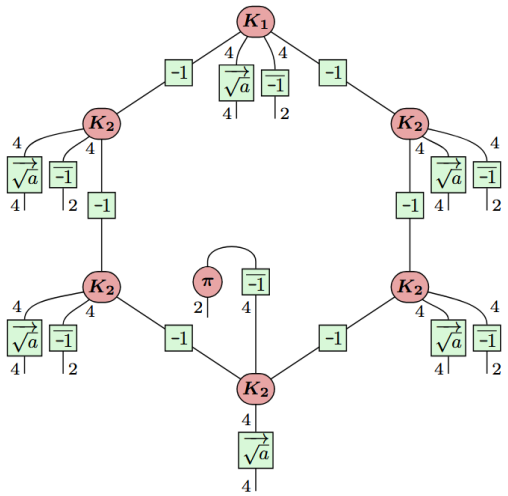

Figure 2: The formal diagrammatic representation of a 2D AKLT state in the Penrose calculus.