Scalable and shallow quantum circuits encoding probability distributions informed by asymptotic entanglement analysis

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Encoding classical data in a quantum state is a key prerequisite of many quantum algorithms. Recently matrix product state (MPS) methods emerged as the most promising approach for constructing shallow quantum circuits approximating input functions, including probability distributions, with only linear number of gates. We derive rigorous asymptotic expansions for the decay of entanglement across bonds in the MPS representation depending on the smoothness of the input function, real or complex. We also consider the dependence of the entanglement on localization properties and function support. Based on these analytical results we construct an improved MPS-based algorithm yielding shallow and accurate encoding quantum circuits. By using Tensor Cross Interpolation we are able to construct utility-scale quantum circuits in a compute- and memory-efficient way. We validate our methods on heavy-tailed distributions important in finance, including on Lévy distributions. We test the performance of the resulting quantum circuits by executing and sampling from them on IBM quantum devices, for up to 64 qubits.

Figures

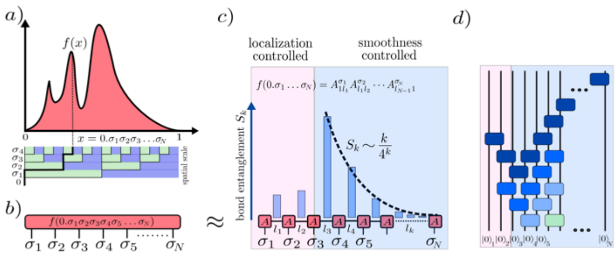

Figure 1: (a) A function f is encoded on a binary grid, with successive digits defining different scales (b) The function is represented as a rank-N tensor. (c) MPS can approximate the tensor, but bond dimensions may a priori be large. We show that for smooth functions the entanglement decays exponentially with scale. For the very large scales the behaviour is non-universal and depends on the localization properties of f.(d) We use these analytical results to construct optimized quantum circuits with linear number of gates, encoding the function f.

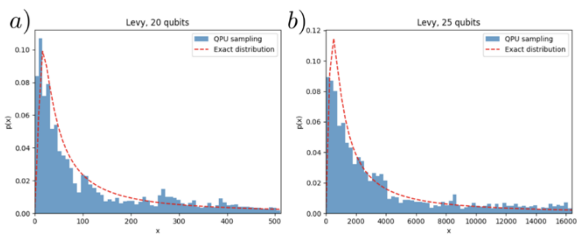

Figure 2: We execute and sample from the encoding quantum circuits constructed using our method on the IBM Torino quantum processing unit (QPU). We consider, among others, the examples of Levy distributions. The samples pass the statistical tests on up to 25 qubits, and the distribution retains qualitatively correct features for u to 64 qubits.

References

[1] Vladyslav Bohun, Illia Lukin, Mykola Lukhanko, Georgios Korpas, Philippe J.S. De Brouwer, Mykola Maksymenko, Maciej Koch-Janusz. arXiv:2412.05202