Strongly Correlated yet Efficient Regimes for Measurement-Based Quantum Computation in Symmetry-Protected Spin Chains

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Resource states for measurement-based quantum computation (MBQC) are known to be exponentially rare in Hilbert space. However, symmetry changes this landscape significantly. In the thermodynamic limit, short-range entangled quantum states organize themselves into what are known as computational phases of quantum matter, driven by symmetry. From a condensed matter perspective, these phases are symmetry-protected topological (SPT) orders. In the context of quantum computation, these phases act as reservoirs of MBQC resource states, where any state within a given SPT phase can perform the same quantum computations. Thus, MBQC power is uniform across all states in an SPT phase.

Quantum states drawn from an SPT phase exhibit two types of entanglement. The first is symmetry-protected entanglement, which remains constant across the entire phase and defines the common characteristics of all states within that phase. The second type is residual "junk" entanglement, which varies throughout the phase and accounts for the differences between individual ground states. In the context of MBQC, the symmetry-protected entanglement is the primary asset, while the residual entanglement presents a significant challenge. The key difficulty in leveraging SPT ground states as MBQC resources lies in mitigating the adverse effects of this residual entanglement.

Regarding the measurements that drive MBQC, a curious dichotomy arises. While symmetry characterizes and classifies computational power, symmetry breaking is required to achieve it. Specifically, MBQC resource states are invariant under the given symmetry, but measurements can enact non-trivial logical gates only if they break that symmetry. This work focuses on the question of how densely algorithmically powerful measurements can be packed into a given resource state.

A priori, there are two regimes: the dilute uncorrelated regime and the dense correlated regime.

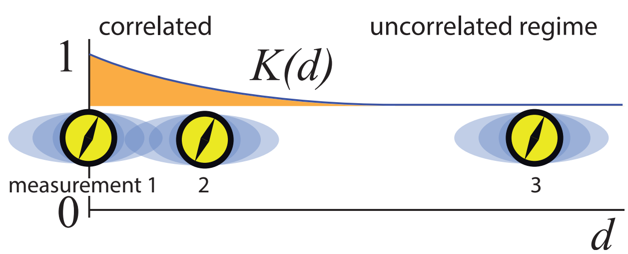

Figure 1: Schematic view of the string order parameter $\mathcal{K}$ vs. distance d; $\mathcal{K}(d)$ is constant for large d. If two local measurements are within a distance where $\mathcal{K}$ still changes, they create a logical effect jointly (correlated regime); otherwise they create separate logical effects (uncorrelated regime).

In the former, the symmetry-breaking local measurements are spaced far apart, whereas in the latter, they are close, relative to a characteristic length scale set by the residual entanglement. The natural inclination has been to avoid the correlated regime. Indeed, all known methods for performing MBQC operate in the uncorrelated regime, where the strategy is to minimize the effects of residual entanglement.

Until now, it has been unclear whether MBQC is even possible in the correlated regime. In this work, we answer this question with an emphatic yes. Moreover, under a general condition on the decay of correlations, we establish that the *strongly correlated regime is actually the most computationally efficient.* In summary, we present a counter-intuitive measurement strategy designed to extract the maximum computational potential from a quantum state drawn from a quantum (SPT) phase.