Quantifying entanglement with global variances

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We introduce witnesses of entanglement based on variances of collective operators, such as the collective spin of ensembles of atoms. These can be seen as generalizations of the well-known *spin-squeezing criterion* [1], which is connected to quantum-enhanced metrological schemes and is often used to quantify entanglement in many-body experiments [2].

We will show how such witnesses can be used for the quantification of entanglement via monotones. First in the case of monotones which can be expressed as minimizations over witnesses [3]. Afterwards, we focus on the so-called *Schmidt-number* or *entanglement dimensionality*, which has been identified as an important resource in quantum information processing, and also as a main obstacle for simulating quantum systems. Its certification is often difficult, and most widely used methods for experiments are based on fidelity measurements with respect to highly entangled states. Here, instead, we generalize the well-known Covariance Matrix Criterion (CMC) [4] to determine the Schmidt number of a bipartite system. This is potentially particularly advantageous in many-body systems, such as cold atoms, where the set of practical measurements is very limited and only variances of collective operators can typically be estimated.

Finally, we show that particular instances of such a general criterion can be evaluated with the use of correlations between measurements in randomized directions [5]. In particular, we find analytical boundary curves for the different entanglement dimensionalities in the space of second- and fourth-order moments of randomized correlations for all dimensions of a bipartite system.

We then show how our method works in practice, also considering a finite statistical sample of correlations, and we also show that it can detect more states than other entanglement-dimensionality criteria available in the literature, thus providing a method that is both very powerful and potentially simpler in practical scenarios.

References

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