

# Theory of non-conserved density accumulations in the anomalous Hall effects

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## Abstract

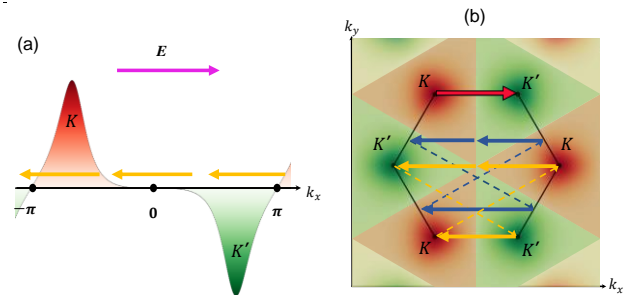
We present a comprehensive theory of the stationary density accumulations that arise in anomalous transport phenomena such as the spin Hall effect, the valley Hall effect, and the orbital Hall effect, where the transported density underlying "charge" (spin, valley imbalance, orbital magnetic moment respectively) is not protected by a conservation law [1]. The theory affords a unified treatment of metals and insulators, topologically trivial or nontrivial, while carefully keeping track of distinct contributions from bulk and edge states as well as under-gap and dissipative currents. We show that the under-gap current does not produce any density accumulation for time-reversal invariant systems: only dissipative currents arising from states at the Fermi surface can do so. We refer to this result as the "no-dissipation-no-accumulation theorem". For time-reversal non-invariant systems, we show that non-dissipative density accumulations can appear (e.g., magnetoelectric polarization) both at the edge and in the bulk. In either case, it is found that the net density accumulation integrated over the sample does not necessarily vanish: as a result of "charge" non-conservation, the density accumulations can have the same sign on both edges of a nanoribbon. In this context, we discuss previous proposals to treat charge non-conservation by a redefinition of the current [2] and show why they are

insufficient. We discuss how the formalism for periodic systems is extended to include the effects of disorder. Finally, we illustrate the formalism by applying it to simple two-dimensional models with edge terminations. In this context, we discuss the peculiar status of the anomalous Hall effect in which the redistribution of the electric charge density, protected by electric charge conservation, allows non-dissipative under-gap currents to coexist with edge density accumulations.

## References

- [1] A. Kazantsev et al., Phys. Rev. Lett 2024 (in press)
- [2] J. Shi et al., Phys. Rev. Lett.96, 076604 (2006)

## Figures



**Figure 1:** Illustration of the no-dissipation-no-accumulation theorem. Panel (a): The cyclic flow of electrons (yellow arrows) in the one-dimensional Brillouin zone of a ribbon subject to the electric field  $E$  (pink arrow). The valley charge changes sign whenever  $k_x$  crosses the boundaries between the red and green regions ( $k_x = 0, \pm\pi$ ). Each electron performs half the cycle as a "left-valley electron" and half as a "right-valley electron". Also shown are the Berry curvature hot spots with positive (negative) value near  $K$  ( $K'$ ). Due to opposite Berry curvatures in the two valleys, the result is a steady valley Hall current. However, in a fully gapped insulator, at the end of the cycle each electron returns to its initial state, thus no valley redistribution occurs and there is no valley density accumulation. Panel (b): Two examples (blue and yellow arrows) of the same flow in the two-dimensional Brillouin zone of the infinite system.