Coarse Graining of Quantum Cellular Automata

Presenting Author: Lorenzo Siro Trezzini
Co-Authors: Paolo Perinotti, Alessandro Bisio.

University of Pavia, Via Bassi, 6, Pavia, Italy

lorenzosiro.trezzini01@universitadipavia.it

Quantum Cellular Automata [1] (QCA) are local dynamics over lattices of quantum systems with discrete time steps. They have many applications, from foundations of quantum mechanics [2] to Many Body Physics. Most notably, they are proven to be a universal model of parallel quantum computation [3]. In particular, they offer promising architecture for the simulation of quantum field theories, as exemplified by the successful implementation of cold atoms on optical lattices or integrated quantum optic simulations.

Given the universality of QCA in computation, a coarse graining procedure would find applications in designing experimental-accessible apparatuses for quantum simulation and computation. Indeed, by neglecting irrelevant degrees of freedom, computation can focus solely on those pertinent for the problem at hand, reducing computational and experimental efforts. We propose a procedure of Coarse Graining largely inspired by Israeli and Goldenfeld's method for classical cellular automata [4].

The main idea is to restrict the dynamics of N steps of a QCA to a subset of 'relevant' degrees of freedom: if this restriction is expressible as a single step of a QCA over the chosen subset, we say that the new automaton is a coarse graining of the original one. This allows us to describe the evolution neglecting irrelevant degrees of freedom. However, with this definition, not all QCA can undergo coarse graining; in general, is not always possible to choose a map that implements the restriction in such a way that what we get behaves like a QCA. We derive a necessary and sufficient condition for a general QCA to be coarse grained relating the original evolution and the restriction map. We then focus on the case of QCA over one dimensional lattices, leveraging a characterization of such QCA in terms of a local topological invariant called the index [5]. This allows to formulate the coarse graining conditions in terms of the structure of the original evolution, facilitating the analysis. Finally, we focus on the case of one-dimensional qubits QCA. This QCA are completely classified [5]. Exploiting this classification, we find all the possible qubit QCA that can be coarse grained. Those are particularly simple, and do not show interesting behavior. However, we have strong reasons to believe that this conclusion is an artifact of the limited number of subspaces within a qubit's cell. Indeed, this strongly constrains the way we can select degrees of freedom.

While this procedure aims to an exact coarse graining, it can also serve as starting point for non-exact methods. Relaxing conditions to allow small simulation errors extends the tool's applicability to general scenarios. Furthermore, beyond practical application, this procedure can shed light on more theoretical questions, as the relation between the renormalization of the simulated field theories and the coarse graining of the underlying computational devices.

References

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