Spectral Properties of Random Clifford Circuits

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Abstract

The Clifford group plays a fundamental role in modern quantum computation and quantum information, because it can be efficiently simulated on classical hardware, and can be augmented to a universal quantum computer by just using additional T-gates [1]. We investigate the spectral properties of random Clifford circuits, U, and that of the corresponding Liouvillian, $L_U \ldots = U \ldots U^+$ [2]. The spectrum of U is in one-to-one correspondence with that of L_U. The latter is in direct relation with the structure and distribution of periodic orbits, i.e., so-called Pauli strings S that are transformed into themselves – apart from a possible sign, the parity of the orbit – after L interations, $(L_U)^n\, S = \tau\, S$.

We build random brick-wall circuits, and sample the closed trajectories (periodic orbits), and determine the distribution of the eigenvalues $\lambda = e^{\,i\,\theta}$ on the unit circle. The distribution $P(\theta) \equiv \langle P_U(\theta)\rangle_U$ can be identified as the autocorrelation of the phases of the eigenvalues of U, and displays peculiar properties: extreme degeneracies as well as some level-repulsion, and has features reminiscent of a fractal pattern.

To investigate the stability of orbits, we introduce $\pi/4$ phase shift gates (T-gates). We find that even a single T-gate completely changes the properties of the circuit [3]. By increasing the number of T-gates, the correlation function rapidly approaches the random matrix theory result, characteristic of random unitary circuits. Nevertheless, some statistically significant fraction of non-trivial orbits persists at low T-gate densities [2].

References

[1] D. Gottesman, Phys. Rev. A **57**, 127 (1998)
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Figures

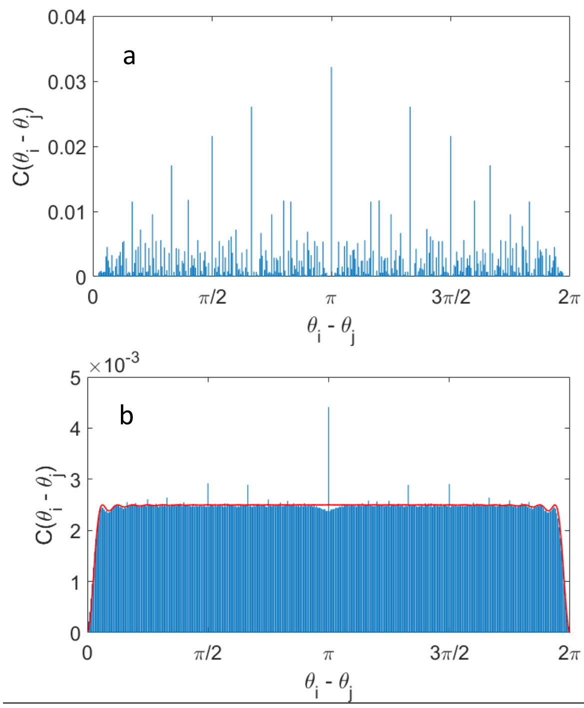

Figure 1: Correlation function of eigenvalue phases $C(\theta_i - \theta_j)$ for random Clifford circuits (a) without and (b) $N_T = 4$ T-gates inserted. The red line corresponds to the analytical result for unitary matrices.