Nonlinear dynamics as a ground-state problem on a quantum computer

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For the solution of time-dependent nonlinear differential equations, we present variational quantum algorithms (VQAs) that encode both space and time in qubit registers. The spacetime encoding enables us to obtain the entire time evolution from a single ground-state computation inspired by the Feynman—Kitaev Hamiltonian [1].
We describe a general procedure to construct efficient quantum circuits using quantum nonlinear processing units [2] for the cost function evaluation required by VQAs. To mitigate the barren plateau problem during the optimisation, we propose an adaptive strategy. The approach is illustrated for the nonlinear Burgers equation. We classically optimise quantum circuits to represent the desired ground-state solutions, run them on IBM Q System One, and demonstrate that current quantum computers are capable of accurately reproducing the exact results.

References

[1] Barison, S. et al. Phys. Rev. Res. 4 (2022), 043161
[2] Lubasch, M. et al. Phys. Rev. A 101 (2020), 010301(R)

Figures

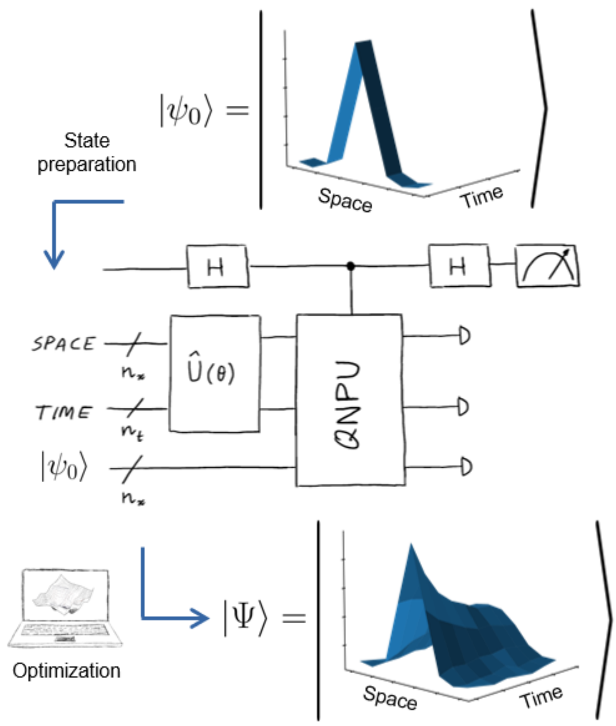

Figure 1: Summary of the procedure

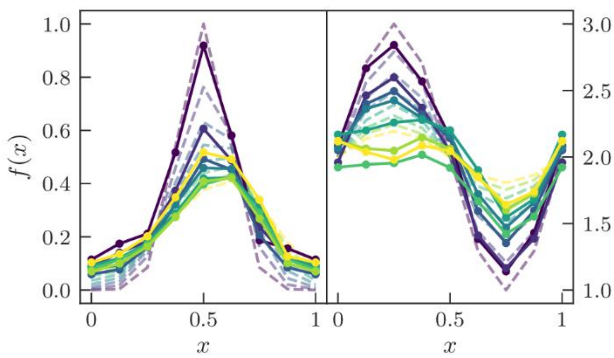

Figure 2: Solutions to the Burgers and diffusion equations on 3 + 3 qubits ($2^3 = 8$ points in time and space) plotted using IBMQ Ehningen (solid lines) and a noiseless simulator (dashed lines). The lighter colours represent increasing time.