Quantum Time Series Similarity Measures and Quantum Temporal Kernels

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Abstract

This article presents a quantum computing approach to the design of similarity measures and corresponding kernels for classification of stochastic symbolic time series. An effective strategy for designing problem-specific kernels is to leverage a generative model of the example space. In this approach, the state space of the generative model serves as the feature space of the kernel. In our study, we use a stochastic quantum generative model known as a Quantum Hidden Markov Model (QHMM) [1] to parameterize the underlying symbolic stochastic process. A QHMM is a Completely Positive Trace-Preserving (CPTP) map defined by a set of Kraus operators associated with observed symbols. Examples are described by the application of corresponding Kraus operators resulting in sequences of mixed quantum states referred to as *generative state sequences*. Sequence similarity is evaluated by appropriate divergence measures within the quantum state space.
We introduce two types of kernels designed for specific classification tasks. Tasks in which the class of a sequence depends on its future stochastic evolution are referred to as *prediction tasks*. Given the assumption of a Markovian process, a sequence's future behaviour depends solely on the final state in its generative state sequence. In such scenarios, the kernel evaluates the sequence similarity using a distance measure between their final generative states. Positive semidefinite distance measures, such as the trace distance, Bures distance, and Frobenius norm are proposed to define these kernels. We refer to these kernels as "predictive kernels".
In another category of tasks, the class of a sequence depends exclusively on its structure, such as the presence of specific patterns. We denote these tasks as 'structural tasks'. In such instances, the kernel maps a sequence to the expectation (or average) of the quantum states in the generative state sequence. This design reflects the assumption that sequence features depend on the full generative state sequence. We refer to these kernels as "structural kernels".
To compare the performance of the proposed kernels against classical ones, we defined classification tasks using a simplified model of directional movements in a stock market. Two common kernel-based algorithms, Support Vector Machine and k-Nearest Neighbours, were implemented with classical and quantum kernels.
In all structural and predictive task scenarios, the quantum kernels exhibited superior performance compared to their classical counterparts. We have implemented predictive quantum kernel on *ibm_nazca* device (Fig. 1).

References

[1] V. Markov et al., "Implementation and learning of quantum hidden Markov models," arXiv:2212.03796 (2022).

Figures

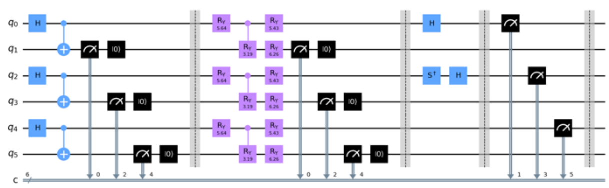

Figure 1: Implementation of projective quantum kernel for market model