Optimizing Bell Inequalities via Tensor Network Contractions in Tropical Algebra

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Abstract

Bell inequalities are natural tools used as witnesses of nonlocality in composite systems. Interestingly, finding the classical bound for a given Bell inequality is in one-to-one correspondence with finding the minimal energy configuration of an associated classical spin system, which is a well-established NP-hard problem.

For Bell inequalities in one-dimensional infinite, translationally invariant (TI) systems with O(1) correlator length, we introduce how to find their corresponding tensors and establish a connection between the notions of tropical eigenvalue and the classical bound per particle as a fixed point of a tropical tensor renormalization procedure [1]. Further, we show the relation between the tropical eigenspaces of the tensor of local deterministic strategies, and their characterization as irreducible domino loops proposed in [2]. This allows us to recover their result in an elegant way, extending it to the finite number of parties case, and for the infinite case as a fixed point of a tropical tensor renormalization procedure. The procedure is also applicable to Bell inequalities with many outcomes such as CGLMP [3], and SATWAP inequalities [4] as a recursive tropical tensor network contraction. Lastly, for the multipartite scenario with any number of inputs and outputs, we show that the upper bound of the number of vertices of the projected Bell local polytope is independent of the number of parties (contrary to e.g. cases with much more symmetry, such as permutational invariance [5,6]) and we provide methods to upper bound its number of vertices.

Our study opens many questions that we leave for future work. In particular, the relation between the tropical eigenspaces of the tensor of local deterministic strategies and the facet structure of the TI local polytope. Also, the study of multipartite nonlocality in systems with e.g. hyperbolic or fractal geometries, which can arise in the context of symmetry-protected topological order [7]. On the numerical side, index slicing and heuristic tensor network contraction orders can improve overall efficiency. The formalism here introduced captures the algorithms based on dynamic programming used for approximating the global optimum of highly nonconvex functions.

References

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