Generalized spin squeezing with limited measurements

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An important and widely used tool in quantum metrology is the spin squeezing parameter. Its development was mainly motivated by two applications: the improvement of precision measurements beyond the classical limit and the study of particle correlations and entanglement [1]. In quantum metrology, the spin squeezing parameter determines the sensitivity that can be achieved through the measurement of a fixed, possibly suboptimal observable. It therefore determines a lower bound on the quantum Fisher information, which expresses the maximal sensitivity achievable with an optimal observable.

However, these sensitivities can only be achieved for an asymptotically large number of measurements. Since these are a limited resource, it is crucial to explore precision bounds in the low data regime. Guided by the recently derived hierarchy of quantum metrology bounds [2], we investigate approximations to generalized quantum information functions beyond the Fisher information that are of relevance in the presence of low data. We present a family of generalized bounds that includes the relation between standard spin squeezing and Fisher information as a particular case.

Our generalized spin squeezing type of bounds are analytically derived from averages and variances of arbitrary measurement observables. We study the families of quantum bounds that may involve higher-order derivatives (Bhattacharyya) and others that avoid the use of differentials altogether (Barankin), as well as combinations of both of them (Hybrid).

We derive analytical expressions for the bounds and for the coefficients that optimize them. For a single qubit, the derived generalized bounds show saturation (see Fig. 1).

References

[1] L. Pezzè et al., *Quantum metrology with nonclassical states of atomic ensembles,* Rev. Mod. Phys. 90, 035005 (2018)

[2] M. Gessner and A. Smerzi, *Hierarchies of Frequentist Bounds for Quantum Metrology: From Cramér-Rao to Barankin*, Phys. Rev. Lett. **130**, 260801 (2023).

Figures

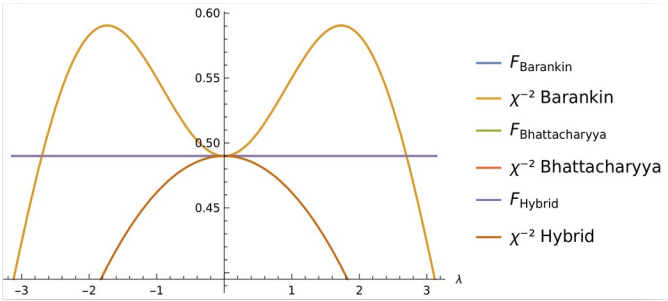

Figure 1: Comparison of the generalized spin squeezing bounds and the generalized quantum information functions that include the Fisher information as a special case. All three families saturate the bound for all values of the single free parameter λ. The Bhattacharyya type of bound coincides with the quantum Cramér-Rao bound for all λ, while the other bounds recover it as λ→ 0.