

# Dual frame optimization for informationally complete quantum measurements

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## Abstract

Randomized measurement protocols such as classical shadows [1] represent powerful resources for quantum technologies, with applications ranging from quantum state characterization and process tomography to machine learning and error mitigation.

Recently, the notion of measurement dual frames, in which classical shadows are generalized to dual operators of POVM effects, resurfaced in the literature [2]. This brought attention to additional degrees of freedom in the post-processing stage of randomized measurements that are often neglected by established techniques.

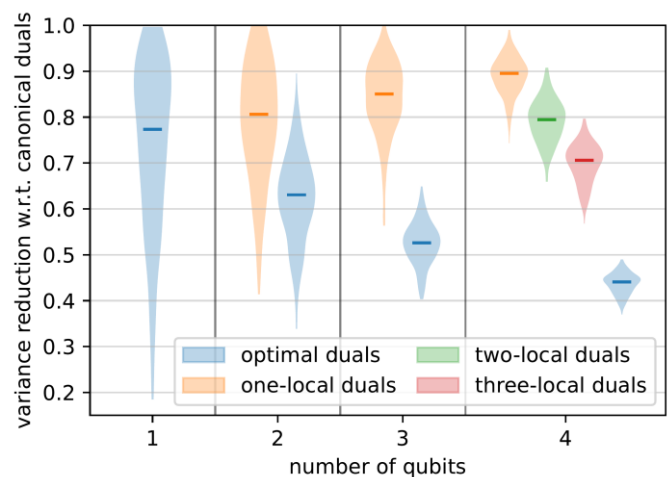
In this work, we leverage dual frames to construct improved observable estimators from informationally complete measurement samples [3]. While standard classical shadows use a canonical dual frame to build observable estimators, we introduce novel classes of dual frames which can be optimized to reduce the variance of expectation value estimations significantly while retaining computational efficiency, see Figure 1.

Remarkably, this comes at almost no quantum or classical cost, thus rendering dual frame optimization a valuable addition to the randomized measurement toolbox.

## References

- [1] Huang, Hsin-Yuan, Richard Kueng, and John Preskill. *Nature Physics* 16, no. 10 (2020): 1050-1057
- [2] Innocenti, Luca, Salvatore Lorenzo, Ivan Palmisano, Francesco Albarelli, Alessandro Ferraro, Mauro Paternostro, and G. Massimo Palma. *PRX Quantum* 4, no. 4 (2023): 040328.
- [3] Fischer, Laurin E., Timothée Dao, Ivano Tavernelli, and Francesco Tacchino. *arXiv preprint arXiv:2401.18071* (2024).

## Figures



**Figure 1:** Variance reduction for the estimation of expectation values compared to standard classical shadows for different types of empirical frequencies dual frames, taken from [3]. Violin plots show the distribution over 200 random pairs of states and observables for the indicated qubit number.