Certifying a complex qubit Hilbert space in a prepare-and-measure scenario: how self-testing helps

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Self-testing is a promising theoretical approach to certifying, e.g., specific quantum states and measurements. Originally, it relied solely on the output statistics of the measurements involved in a device-independent setup. We turn to a semi-device-independent setup by considering a prepare-and-measure scenario for qubit messages.

We construct linear witnesses W (i.e., linear functionals on the outcome statistics of the measurements) for performing self-tests in possibly minimal setups within this scenario. In a setup involving four (three) preparations and three (two) projective measurements in addition to the target, we exemplify how to self-test any four- (three-) outcome extremal positive operator-valued measure. We also achieve self-testing of any number of pure state preparations with the help of three projective measurements at most.

However, self-testing is a purely theoretical tool, since it requires reaching the theoretical quantum maximum $W_{\mathbb{C}^2}$ of the witness by the outcome statistics. A relaxation is to certify only a single but well-defined property of the preparations or the relevant measurement.

In this spirit, we provide a means to certify that a set of four prepared states spans a complex qubit Hilbert space. For this purpose, we introduce a specific configuration of four complex qubit states, which we call umbrella-like and which depends on a real parameter $c\in[0,3)$, and design a linear witness for self-testing this set in a minimal setup.

We then determine, for any given c, the maximum value $W_{\mathbb{R}^2}$ attainable by evaluating the so-defined witness for any configuration of four real qubit preparations. Then should we perform an experiment and evaluate the witness (for a c value of our choice) to exceed $W_{\mathbb{R}^2}$, the complex nature of the Hilbert space spanned by the actual preparations becomes certified. We expect such a certification scheme to be useful for preparations that are intended to implement the set of states targeted by the design of the witness but which are subject to experimental noise.

To illustrate the idea experimentally, we implemented prepare-and-measure scenarios for self-testing umbrella-like configurations as quantum circuits on publicly available quantum processors of IBM and IonQ. Experimental witness values exceeded $W_{\mathbb{R}^2}$ for most values of c (Fig. 1.). Thereby, we successfully certified the existence of a complex qubit Hilbert space in our experiments.

Figures

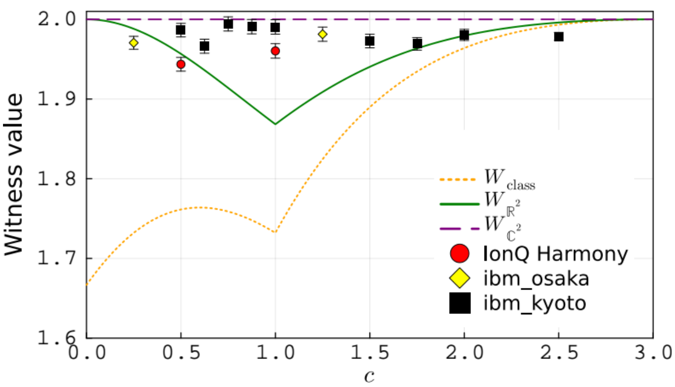

Figure 1: The witness W designed to self-test umbrella-like configurations, as a function of the umbrella parameter c. The maximum values attainable using classical strategies (W_{class}), real qubits ($W_{\mathbb{R}^2}$), and complex qubits ($W_{\mathbb{C}^2}$) are presented. The best experimentally measured values are also displayed for specific values of c.