Additive twisted codes: new distance bounds and infinite families of quantum codes

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Quantum error-correcting codes, or simply quantum codes, are used to protect quantum information from corruption by noise (decoherence) on the quantum channel, much like classical error-correcting codes. The most common approach to constructing quantum codes is through the stabilizer formalism, which establishes a connection between certain dual-containing additive codes and quantum (stabilizer) codes [1, 2].

One of the main challenges in designing quantum stabilizer codes is the dual-containment (commuting) condition, which restricts the use of classical codes in constructing good quantum codes. In this talk, we first present a construction method for binary stabilizer quantum codes that allows the utilization of additive codes that are not dual-containing (equivalently, containing a non-commuting set of generators).

Next, we focus on a family of classical codes called additive twisted codes for constructing binary quantum codes. We establish a stronger connection between twisted codes and linear cyclic codes, enabling us to derive novel minimum distance lower and upper bounds for twisted codes and identify new similarities between twisted codes and linear cyclic codes. In particular, we prove that the Hartmann-Tzeng bound holds for twisted codes.

Finally, we present five infinite families of record-breaking and sometimes optimal binary quantum codes that can be constructed from twisted codes using these bounds. To determine whether a quantum code is record-breaking and/or optimal, we refer to the tables maintained by Markus Grassl [3]. A more detailed explanation of the material covered in this talk can be found in [4, Chapter 3].

References

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