# Role of Lifshitz transitions and Berry curvature dipole on nonlinear Hall effect in low symmetry Bilayer graphene 

## Tanweer Ahmed ${ }^{\text {a }}$

Bao Q. Tua, Takashi Taniguchib, Kenji Watanabe ${ }^{\text {b }}$, Marco Gobbic,d , Félix Casanovaa,d and Luis E. Huesoa,d
${ }^{a}$ CIC nanoGUNE BRTA, 20018 Donostia-San Sebasti'an, Basque Country, Spain.
${ }^{b}$ National Institute for Materials Science, 1-1 Namiki, Tsukuba 305-0044, Japan.
${ }^{\text {c Centro }}$ de Física de Materiales (CSIC-EHU/UPV) and Materials Physics Center (MPC), 20018 Donostia-San Sebastián, Basque Country, Spain dIKERBASQUE, Basque Foundation for Science, 48009 Bilbao, Basque Country, Spain
t.ahmed@nanogune.eu

Abstrac $\dagger$
The second harmonic ( $2 \omega$ ) nonlinear Hall effect (NLHE) [1,2] is both technologically relevant ${ }^{2}$ and fundamentally important. On one hand, it can potentially bring a paradigm shift in logic and energyharvesting technologies by replacing the age-old interface-based devices with their bulk crystal-based counterparts [3]. On the other hand, it provides rich information on the locations of saddle points [4] and flat bands and directly probes topological phase transitions in atomically thin Chern insulators ${ }^{[5]}$. Obtaining such information on electronic properties is crucial in the case of heterostructures of atomically thin quantum materials, where structural symmetry engineering and thermodynamically tunable complex quasiparticle bands coexist. In this work, we experimentally study NLHE on inversion symmetry broken high-quality bilayer graphene (BLG) as a function of doping (n) dielectric displacement filed (D) and temperature (T). Our results reveal an unforeseen duopoly of extrinsic scattering and interfacial strain-induced intrinsic Berry curvature dipole (BCD), whose sign and magnitude can be tuned by $n$ and/or $D$
near the low energy band edge of BLG. The second harmonic generation efficiency $V_{x x(y)^{2 \omega}} / V_{x x}{ }^{\omega}{ }^{2}$ in BLG is $\sim 50 V^{-1}$, highes $\dagger$ among all scalable materials. Moreover, $n$ - D dispersion of the sign change of $V_{x x(Y)^{2 \omega}}$ traces out the topologically relevant Lifshitz transitions in BLG. Our work establishes BLG as a highly tunable platform to generate NLHE, which in turn probes the fascinating low-energy electronic structure in Bilayer graphene.

## References

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Figures


Figure 1: n-D phase space of $V_{x x}{ }^{2 \omega} / V_{x x} \omega$ 2 collected from one of our samples. The dashed lines indicate sign changes at charge neutrality and at Lifshitz transitions.

