Input-output theory for quantum circuits based on hierarchical equation of motion

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Hierarchical equation of motion (HEOM) [1] is known as one of the most popular and reliable methods of modelling of open quantum system dynamics. This technique can be straightforwardly applied for a broad class of Gaussian environments. Free-pole expansion [2] allows to extend this method for environments sub-Ohmic spectral densities and drastically enhance its performance even in low temperature limit.
HEOM represents a first order in time differential equation for the reduced density operator of the system and so-called auxiliary density operators which contain information about system–environment correlations. The latter are usually disregarded in the end since it is thought that all the system–related observables can be calculated using just reduced density operator. However, for a typical example of open quantum systems, namely superconducting circuits coupled to transmission lines, the only quantities accessible to experimentalists are output fields in these lines, which are observables of the environment rather than of the system. For systems in Markovian environments the solution to this problem is given by input–output theory [3,4] which relates system observables to the output fields. For linear circuits the scattering problem can be solved using just classical approach.
In our work we show that the HEOM can serve as a basis for construction of both non-Markovian and nonlinear input–output theory. Auxiliary density operators play a key role in this theory, since they contain all the necessary information about system–environment correlations. We illustrate our theory on the readout of a recently developed superconducting qubit called unimon [5].

References

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