# Geometry and Faddeev-Jackiw quantization of electrical networks

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Abstract: In lumped-element electrical circuit theory, the problem of solving Maxwell's equations in the presence of media is reduced to two sets of equations. Those addressing the local dynamics of a confined energy density, the constitutive equations, encapsulating local geometry and dynamics, and those that enforce the conservation of charge and energy in a larger scale that we express topologically, the Kirchhoff equations. Following a consistent geometrical description, we develop a new and systematic way to write the dynamics of general lumped-element electrical circuits as first order differential equations derivable from a Lagrangian and a Rayleigh dissipation function. Combining this construction with the Faddeev-Jackiw[1] method, we are able to identify and classify all singularities that arise in the search for Hamiltonian descriptions of general networks. Furthermore, we provide systematics to solve those singularities, which is a key problem in the context of canonical quantization of superconducting circuits[2,3]. The core of our solution relies on the correct identification of the reduced manifold in which the circuit state is expressible, e.g., a mix of flux and charge degrees of freedom, including the presence of compact ones. We apply the fully programmable method to obtain (canonically quantizable) Hamiltonian descriptions of nonlinear and nonreciprocal circuits which would be cumbersome/singular if pure node-flux or loop-charge variables are used as a starting configuration space. Generalizations beyond the lumped-element approximation as continuous limits thereof, and a rigorous quantum description of dissipation are discussed. This work unifies diverse existent geometrical pictures of electrical network theory, and will prove useful, for instance, to automatize the obtention of exact Hamiltonian descriptions of superconducting quantum chips.

## References

[1] L. Faddeev and R. Jackiw, Physical Review Letters **60** (1988) 1692.

[2] I. L. Egusquiza and A. Parra-Rodriguez, Physical Review B **106** (2022) 024510.

[3] M. Rymarz and D. DiVincenzo, arXiv:2208.11767 (2022).

## Figures

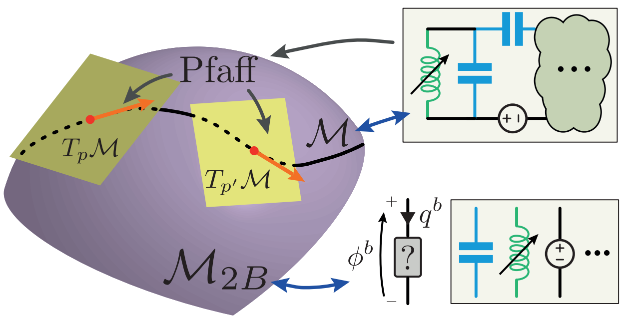


**Figure 1:** Artistic geometrical picture of the state of a collection of lumped elements. There are two variables per branch (flux $\phi^b$ and charge $q^b$) that span the manifold $\mathcal{M}_{2B}$. Connecting the branches in a circuit gives rise to constraints, collected in a Pfaff system that fixes the possible directions at each point on $\mathcal{M}_{2B}$. The integration of those directions results in the integral (reduced) manifold $\mathcal{M}$ that describes the constrained physics.