Towards polynomial convergence for Variational Quantum Algorithms using Langevin Dynamics

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One of the most promising types of algorithms to run on noisy intermediate-scale quantum computers are variational optimization algorithms [4]. In those algorithms one deals with a parametrized quantum circuit whose outputs are then a parametrized family $\mathcal{F}$ of n-particle quantum states.
Given an n-body observable H, that can be efficiently implemented (e.g. a locally interacting Hamiltonian), the goal is to obtain an approximation of

$$\min_{|\psi\rangle\in\mathcal{F}} F(|\psi\rangle) = \langle\psi|\mathrm{H}|\psi\rangle. \qquad (1)$$

The goal of this paper is to study the continuous Langevin Dynamics (see [1, 2]) in a rather general setting, shown in Figure 2. Proving convergence results in such a setting may potentially lead to poly-time algorithms to solve (1) on depth-2 quantum circuits with gates acting on $\log L$ sites, with L being the system size (Figure 1). Moreover, our results should be applicable to other circuits, under certain assumptions on

$$F(|\psi\rangle) = \langle\psi|\mathrm{H}|\psi\rangle.$$

We generalize [3] to the Lie Group $SU(n)$; we prove that for any values $\varepsilon, \delta \in (0,1)$, for

$$\beta \geq \Omega\left(\frac{d^2 \log d}{\varepsilon} - \frac{\log \delta}{\varepsilon}\right)$$

we get that the Gibbs distribution ν associated to our Markov process satisfies

$$\nu(F - \min_{y\in SU(n)^{\times r}} F(y) \ \geq\ \varepsilon) \leq \delta.$$

Furthermore, we prove that our setting satisfies a logarithmic Sobolev Inequality, which guarantees exponential convergence of the process Z_t to ν (see [1]).

References

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Figures

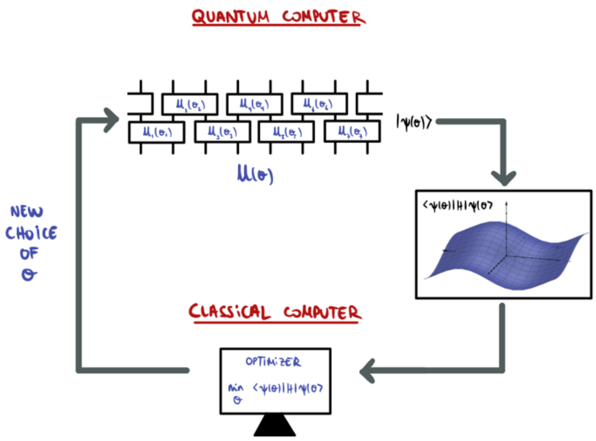

Figure 1: General Variational Quantum Algorithm Scheme.

$$\min_{x\in SU(n)^{\times r}} F(x), \quad F : SU(n)^{\times r} \to \mathbb{R} \text{ non-convex},$$

$$dZ_t = -grad F(Z_t)\, dt + \sqrt{\frac{2}{\beta}}\, dW_t.$$

Figure 2: Langevin Dynamics proposed to find the minimum of F.