Quantum Fractals

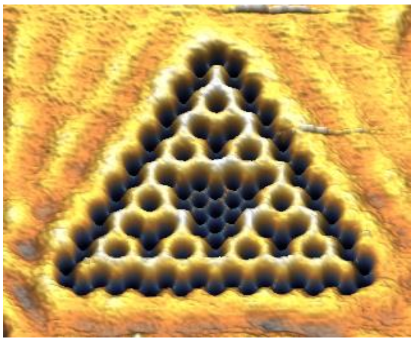
Cristiane Morais Smith

Institute for Theoretical Physics, Utrecht University, The Netherlands

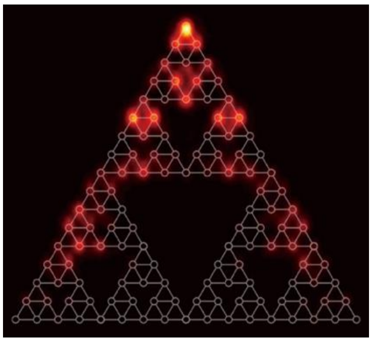
C.deMoraisSmith@uu.nl

The human fascination for fractals dates back to the time of Christ, when structures known nowadays as a Sierpinski gasket were used in decorative art in churches. Nonetheless, it was only in the last century that mathematicians faced the difficult task of classifying these structures. In the 80's and 90's, the foundational work of Mandelbrot triggered enormous activity in the field. The focus was on understanding how a particle diffuses in a fractal structure. However, those were classical fractals.

This century, the task is to understand quantum fractals. In 2019, we realized a Sierpinski gasket using a scanning tunneling microscope to pattern adsorbates on top of Cu(111) and showed that the wavefunction describing electrons in a Sierpinski gasket fractal has the Hausdorff dimension d = 1.58 [1]. However, STM techniques can only describe equilibrium properties.



Last year, we went a step beyond and using photonics experiments we unveiled the quantum dynamics in fractals. By injecting photons in waveguide arrays arranged in a fractal shape, we we able to follow their motion and understand their quantum dynamics with unprecedented detail. We built 3 types of fractal structures to reveal not only the influence of different Hausdorff dimension, but also of geometry [2].



The investigation of systems living in non-integer dimensions raises the question whether fractional calculus might be a useful concept. In the last part of this talk, I will discuss a system described by a fractional Langevin equation with white noise [3]. Fractional calculus is shown to connect different states of matter and has revealed the elusive time glass phase previously conjecture by Frank Wilczek

References

[1] S.N. Kempkes, M.R. Slot, S.E. Freeney, S.J.M. Zevenhuizen, D. Vanmaekelbergh, I. Swart, and C. Morais Smith, "Design and characterization of electronic fractals", Nature Physics 15, 127(2019) [see also 15 years of Nature Physics, Nature Physics 16, 999 (2020)].
[2] X.-Y. Xu, X.-W. Wang, D.-Y. Chen, C. Morais Smith, and X.-M. Jin, "Quantum transport in fractal networks," Nature Photonics 15, 703 (2021).
[3] R. C. Verstraten, R. F. Ozela, and C. Morais Smith, "Time glass: a fractional calculus approach", Phys. Rev. B 103, L180301 (2021).