Fragmented imaginary-time evolution for intermediate-scale quantum signal processors

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Simulating quantum imaginary-time evolution (QITE), that is, implementing the operator $F_\beta(H) = e^{-\beta H}$ for a given Hamiltonian H and an imaginary time $-i\beta$, is a major promise of quantum computation. QITE is an important subroutine for many quantum algorithms, such as ground-state optimisations, partition-function estimation, and quantum Gibbs-state sampling. However, the known state-independent algorithms are either probabilistic (repeat until success) with unpractically small success probabilities or coherent (quantum amplitude amplification) but with circuit depths and ancillary-qubit numbers unrealistically large for the mid term. Our main contribution is a new generation of deterministic, high-precision QITE algorithms significantly more amenable to intermediate-scale quantum devices [1]. These are based on a surprisingly simple idea: partitioning the evolution into several fragments that are sequentially run probabilistically, as shown in Fig. 1. This causes a huge reduction in wasted circuit depth every time a run fails. Indeed, the resulting overall runtime is asymptotically better than in coherent approaches and the hardware requirements even milder than in probabilistic ones, remarkably, as exemplified in Fig. 2.
On a more technical level, we present two QITE-circuit sub-routines with excellent complexity scalings. One of them is optimal in ancillary-qubit overhead (one single ancillary qubit throughout) whereas the other one is optimal in runtime for small inverse temperature or high precision. The latter is shown by noting that the runtime saturates a cooling-speed limit that is the imaginary-time counterpart of the celebrated no fast-forwarding theorem of real-time simulations, which we prove. Moreover, we also make a technical contribution to the quantum signal processing formalism [3,4] (on which our subroutines are based) for operator-function synthesis from their Fourier expansion that is useful beyond QITE.
Our results are relevant to near-term quantum hardware. In particular, they constitute a versatile toolbox for the demonstration of experimental quantum signal processors.

Figures

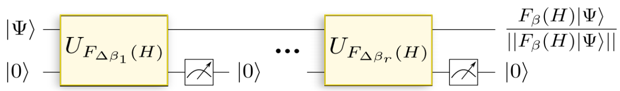

Figure 1: Schematic representation of fragmented QITE algorithm where several fragments are implemented probabilisticaly. For each $\Delta\beta$, after post-selecting the ancilla in the state $|0\rangle$ one has $\langle 0|U_{F_{\Delta\beta}(H)}|0\rangle \propto F_{\Delta\beta}(H)$.

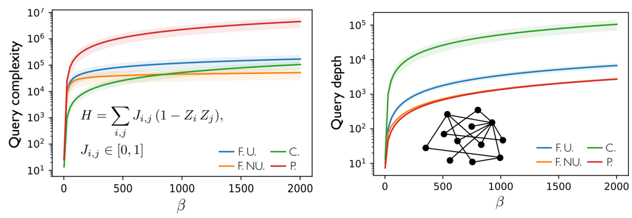

Figure 2: Results: comparison of the total number of queries to a Hamiltonian oracle and circuit depth for different algorithms for the weighted max-cut model. C. (coherent), P. (probabilistic), F.U. and F. NU (fragmented with different imaginary-time fragments).

References

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[3] A. Gilyén et al; STOC 2019 (2019), 193.