Uncertainty relations with the variance and the quantum Fisher information based on convex decompositions of density matrices

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We present several inequalities related to the Robertson-Schrödinger uncertainty relation [1]. In all these inequalities, we consider a decomposition of the density matrix into a mixture of states, and use the fact that the Robertson-Schrödinger uncertainty relation is valid for all these components. By considering a convex roof of the bound, we obtain an alternative derivation of the relation in Ref. [2], and we can also list a number of conditions that are needed to saturate the relation. We present a formulation of the Cramér-Rao bound involving the convex roof of the variance. By considering a concave roof of the bound in the Robertson-Schrödinger uncertainty relation over decompositions to mixed states, we obtain an improvement of the Robertson-Schrödinger uncertainty relation. We consider similar techniques for uncertainty relations with three variances. Finally, we present further uncertainty relations that provide lower bounds on the metrological usefulness of bipartite quantum states based on the variances of the canonical position and momentum operators for two-mode continuous variable systems. We show that the violation of well-known entanglement conditions in these systems discussed in Refs. [3,4] implies that the state is more useful metrologically than certain relevant subsets of separable states. We present similar results concerning entanglement conditions with angular momentum operators for spin systems

References

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