Studying the Limiting Behaviour of Qubit Hamiltonians Using Graphons

Amy Searle[1]
Joseph Tindall[1], Abdulla Alhajri[1]
[1]*Clarendon Laboratory, The University of Oxford, Oxford, OX13PU, United Kingdom*
amy.searle@physics.ox.ac.uk

Although qubit Hamiltonians are usually studied on lattice structures, recently there has been interest in studying the behaviour of such systems on top of more general graph structures [1,2](see Figure 1), such as random graphs. Usually it is properties in the thermodynamic limit which are of interest, so that the behaviour of the system on graphs of increasing size, or a graph sequence, is studied in order to discern its limiting behaviour. These calculations are necessarily limited by current state-of-the-art many body simulations [3]. On the other hand, the graph sequence itself is known to have a unique limit, known as a *graphon* [4].

Here, we define a system of qubits with XXZ Hamiltonian on the graphon directly. We find that Hamiltonians resulting from different graph sequences can be identical, up to a rescaling. For example, the Erdös Renyi graphon (see Figure 2) has the same Hamiltonian as the fully symmetric graphon, and for the latter it can be analytically shown that all qubits behave as a single collective unit and no many-body physics arises. More generally, the automorphism group of the graphon constrains the possible ground states, so that systems with graphons with relatively few spatial symmetries can display more interesting and complex behaviour. This furthers results obtained in [5] by the present authors using tensor network simulations.

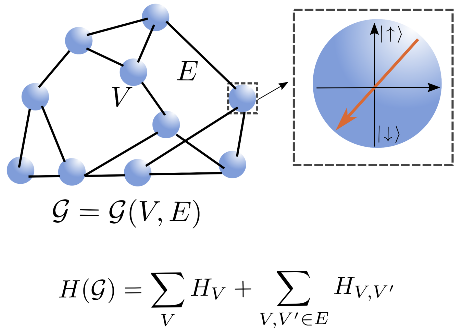

Figure 1: Defining a system of interacting qubits over a graph with vertex set V and edge set E. Each node represents a qubit and qubits share an interaction term $H_{V,V'}$ in the Hamiltonian if and only if the graph has an edge between vertices V and V'. (reproduced from [5])

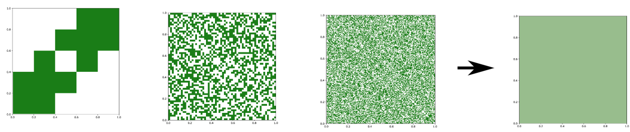

Figure 2: The graphon as a graph limit. The large squares, from left to right, correspond to a sequence of adjacency matrices of Erdös Renyi graphs for increasing N and $p = 0.5$; the rightmost square corresponds to the graph limit, the graphon, $W(x, y) = 0.5$, for $x,\ y \in [0,\ 1]$.

References

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