Quantum Algorithms for Solving Problems in Orbital Mechanics

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Orbital Mechanics represents the application of celestial mechanics to spacecraft motion. In this context, complex configurations (e.g., terminal Rendezvous, formation flying, N-body systems) are usually described by differential equations (DEs) [1]. Progress has been achieved in the solution of such problems, with evidence that quantum algorithms provide a substantial speedup over classical numerical methods [2]. Hence, this research focuses on solving DEs via two quantum algorithms - based on the truncated Taylor Series ("TS algorithm") [3] and variational circuits ("VC algorithm") [4], respectively.

We have applied the TS algorithm to the linearized equations of the relative motion (Hill equations) [1]. After solving the DEs analytically, we write the correspondent k-order Taylor Series and encode it into the quantum circuit implemented in Yao Quantum [5]. Our answer's convergence (compared to the exact solution) has been analyzed as a function of the truncated Taylor Series order (k) and the number of qubits in the circuit (Figure 1a).

To solve the nonlinear differential equations of the relative motion, we have used the VC algorithm implemented in Pennylane [6]. Given the initial conditions and hyperparameters (arguments of the quantum gates), the circuit is trained by classical optimizers to provide an output closer to the desired function. We then studied the model's convergence as a function of intrinsic parameters of the variational circuit and the number of qubits. Furthermore, we compare our output to the numerical solution of the problem (Figure 1b). Thus, the present work constitutes a proof of concept for modeling Orbital Mechanics problems with quantum algorithms.

References

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Figures

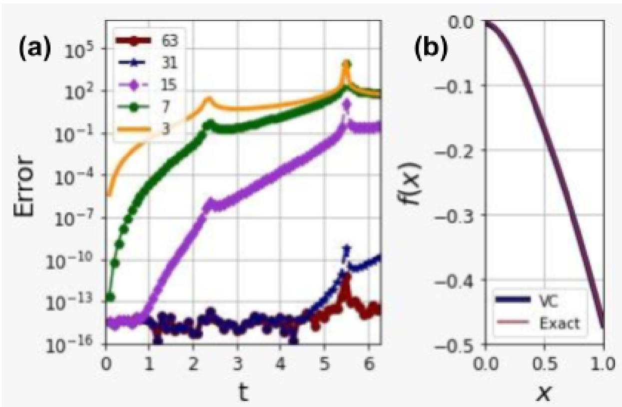

Figure 1: (a) Error as a function of the Taylor Series order (k). (b) Comparing the solutions (VC and exact) of the nonlinear DE.