Extrapolation to the complete basisset limit in density-functional theory using statistical learning

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The numerical precision of density-functional-theory (DFT) calculations depends on a variety of computational parameters, one of the most critical being the basis-set size [1, 2, 3]. The ultimate precision is reached with an infinitely large basis set. i.e., in the complete basis-set (CBS) limit. Our aim is to find a machine-learning model that extrapolates finite basis-size calculations to the CBS limit. Quantile random forests (QRF) and symbolic regression, applying the SISSO approach, are used to estimate total energies, lattice parameters, and band gaps as a function of the basis-set size. The QRF model outperforms previous approaches in the literature for both codes, while SISSO outperforms the random-forest model for the exciting code [4]. Our approach also provides prediction intervals, which quantify the models' uncertainty. Uncertainty estimates for the total energies would provide users useful information about the precision of these calculations and how the data can be reused/repurposed [5]. We evaluate our work on datasets consisting of 63 binary solids and 3000 binary semiconductors, respectively.

References

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Figures



Figure 1. Predictions of total-energy differences to the CBS of the QRF model on FHI-aims (left) and exciting (right) data (63 binaries of different basis-set size precision), plotted against the respective DFT results. Relevant error metrics are added to help interpret the quality of the fit. Note the logarithmic axes. The region of DFT calculated ΔE^{AB} values between 1 meV/atom and 1 eV/atom is plotted with a lighter shade since these data are of particular interest to DFT practitioners.