Birth Quota of Non-Generic Degeneracy Points

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Weyl points are generic and stable features in the energy spectrum of Hamiltonians that depend on a three-dimensional parameter space. Non-generic isolated two-fold degeneracy points, such as multi-Weyl points [1], split into Weyl points upon a generic perturbation that removes the fine-tuning or protecting symmetry. The number of the resulting Weyl points is at least $|Q|$, where Q is the topological charge associated to the non-generic degeneracy point. Here, we show that such a non-generic degeneracy point also has a birth quota, i.e., a maximum number of Weyl points that can be born from it upon any perturbation [2]. The birth quota is a local multiplicity associated to the non-generic degeneracy point, an invariant of map germs known from singularity theory [3]. This holds not only for the case of a three-dimensional parameter space with a Hermitian Hamiltonian, but also for the case of a two-dimensional parameter space with a chiral-symmetric Hamiltonian. We illustrate the power of this result for band structures of two- and three-dimensional crystals. Our work establishes a strong and powerful connection between singularity theory and topological band structures, and more broadly, parameter-dependent quantum systems.
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References

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[2] György Frank et al, arXiv:2202.05825 (2022)
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Figures

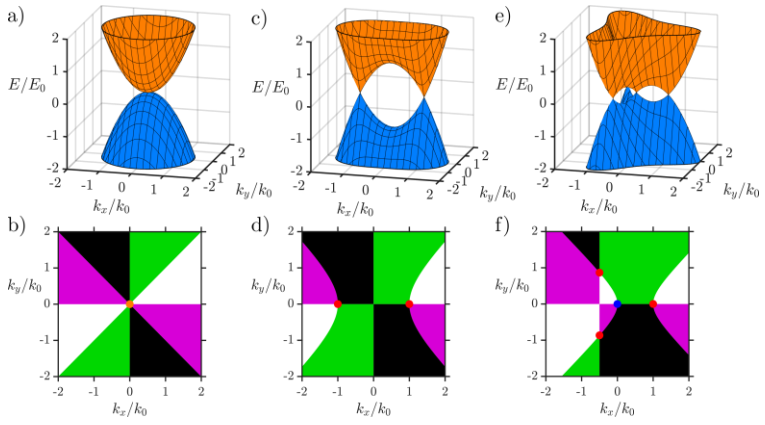

Figure 1: Birth of 2D Weyl points from a non-generic degeneracy point in bilayer graphene. a,b) Non-generic degeneracy point. c,d) Two 2D Weyl points born due to mechanical strain. e,f) Four 2D Weyl points born upon due to trigonal warping. a,c,e) Dispersion relation of the valence and conduction bands. b,d,f) Colored plane shows the sign structure of the effective Hamiltonian map of bilayer graphene.