Centre for Advanced 2D Materials

NUS National University of Singapore

Theory of Coulomb drag in spatially inhomogeneous graphene

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For the details, see **arXiv:1611.03089v2**

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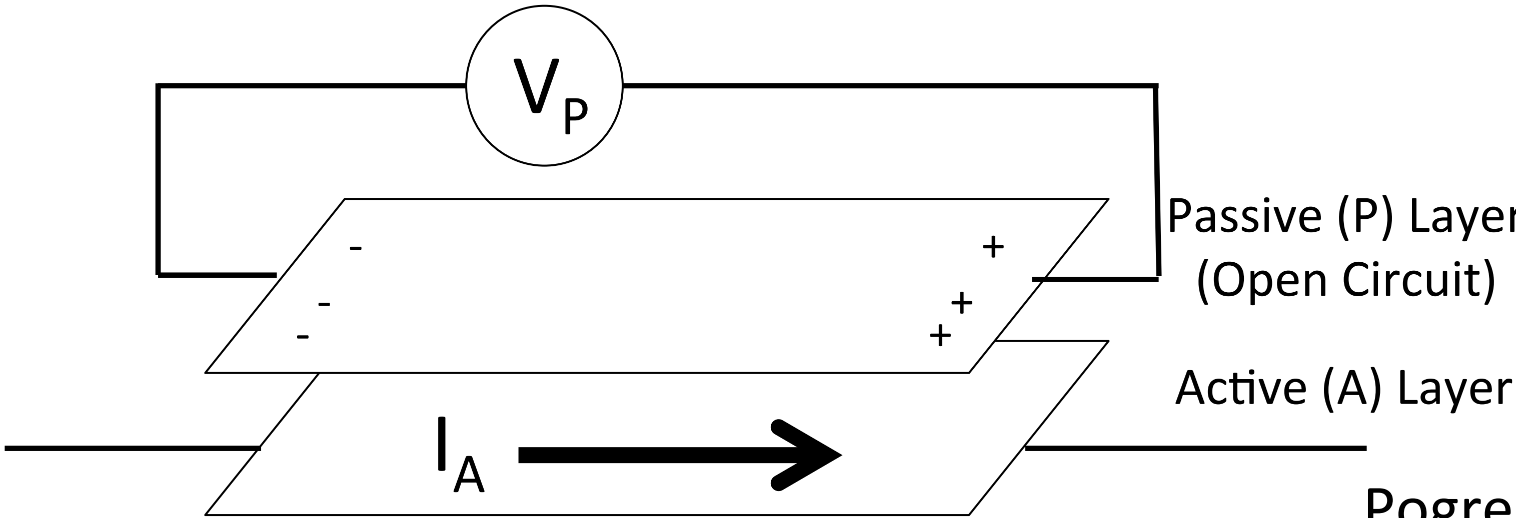
The Coulomb Drag Effect



Pogrebinskii
(1977)

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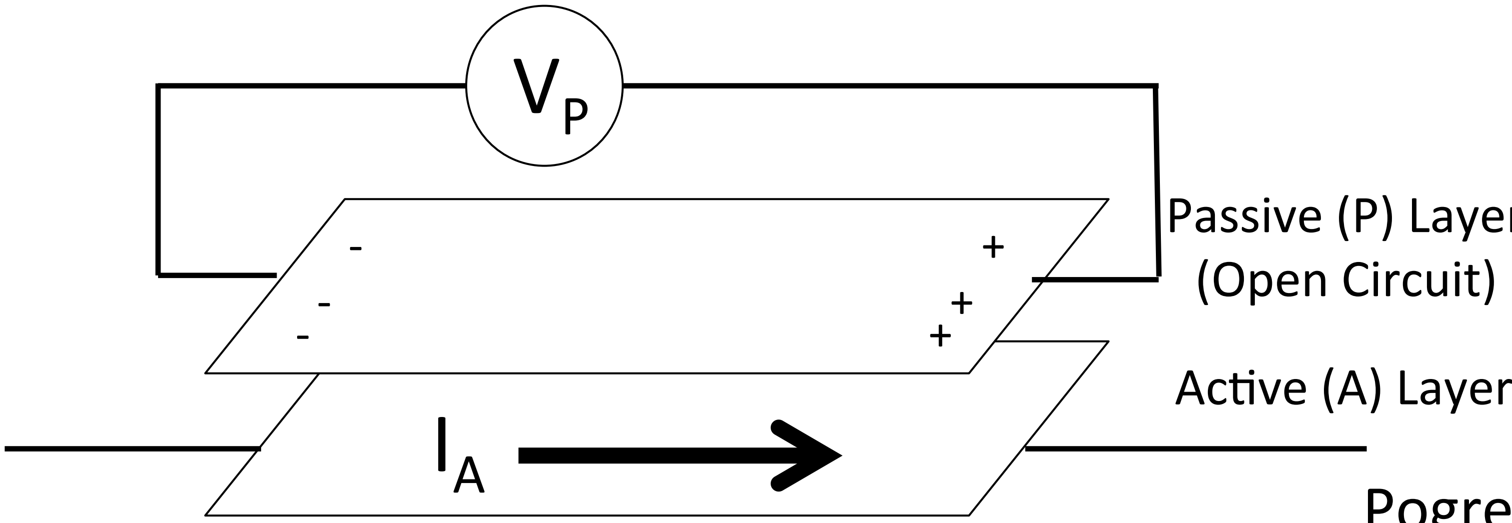
The Coulomb Drag Effect



Pogrebinskii (1977)

Quantified by Drag Resistivity:

$$\rho_{\mathrm{D}} \propto \frac{V_{\mathrm{P}}}{I_{\mathrm{A}}} \quad \Rightarrow \quad \rho_{\mathrm{D}} = -\frac{\sigma_{\mathrm{D}}}{\sigma_{\mathrm{A}}\sigma_{\mathrm{P}} - \sigma_{\mathrm{D}}^2}$$

Why is it important/interesting?

1. Measures pure electron-electron interaction.

2. New and interesting physics.
- Integer and fractional Hall <u>drag</u> effects
- Long-lived indirect excitons
- Excitonic superfluidity

3. Smoking gun experiment for the indirect exciton condensate

4. Renewed interest in Coulomb drag with new 2D materials.

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Why graphene?

1. Electrons confined to a 2D atomic plane.

2. Unprecedentedly small layer separations ~ 1 nm allow for strong interlayer interactions.

3. Tunable charge densities allow exploration of both high and low density regimes.

4. Other 2D materials (TI's, black phosphorous, etc) are of interest too.

Lots of theoretical work has been done

Parabolic 2D Electron Gas

1. Jauho & Smith, PRB (1993)
2. Vignale & MacDonald, PRL (1996)
3. Many more…

Dirac Fermions

1. Tse et al, PRB (2007)
2. Narozhny et al, PRB (2012)
3. Many more…

$$\sigma_D = \frac{1}{16\pi k_B T} \int_{-\infty}^{\infty} \frac{d^2q}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{d\omega}{\sinh^2(\frac{\hbar\omega}{2k_B T})} \Gamma^x(\omega, \mathbf{q}, \frac{\mu_A}{k_B T}) \Gamma^x(\omega, \mathbf{q}, \frac{\mu_P}{k_B T}) |V(q, \omega, d)|^2$$

$$\rho_{\mathrm{D}} = -\frac{\sigma_{\mathrm{D}}}{\sigma_{\mathrm{A}}\sigma_{\mathrm{P}} - \sigma_{\mathrm{D}}^2}$$

'Momentum' drag

- Our motivation: How well does this standard theory agree with graphene experiment?

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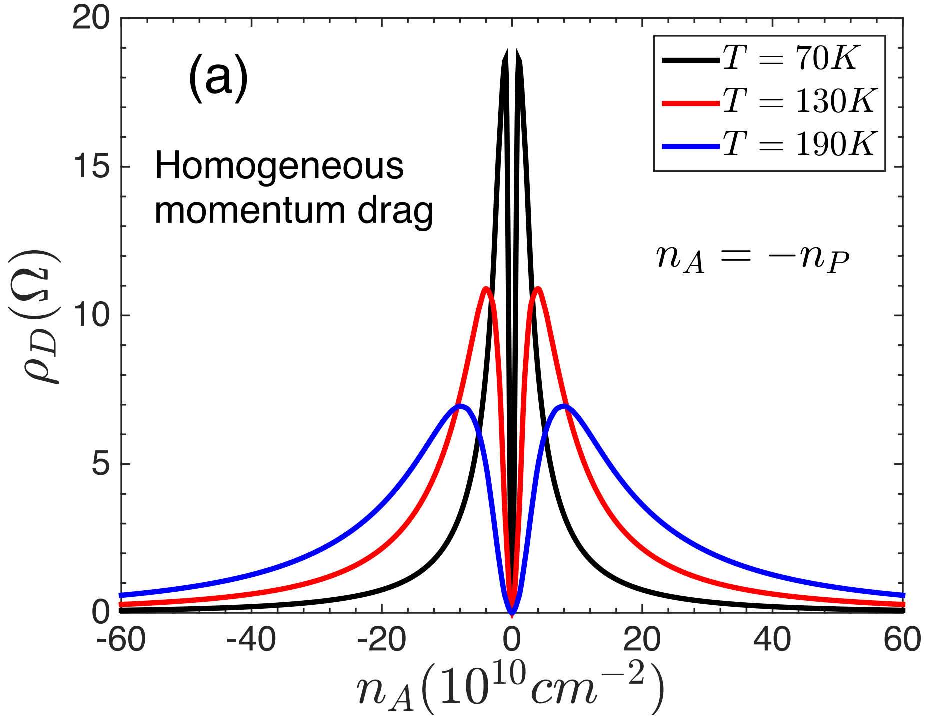
'Momentum' drag

- Our motivation: How well does this standard theory agree with graphene experiment?

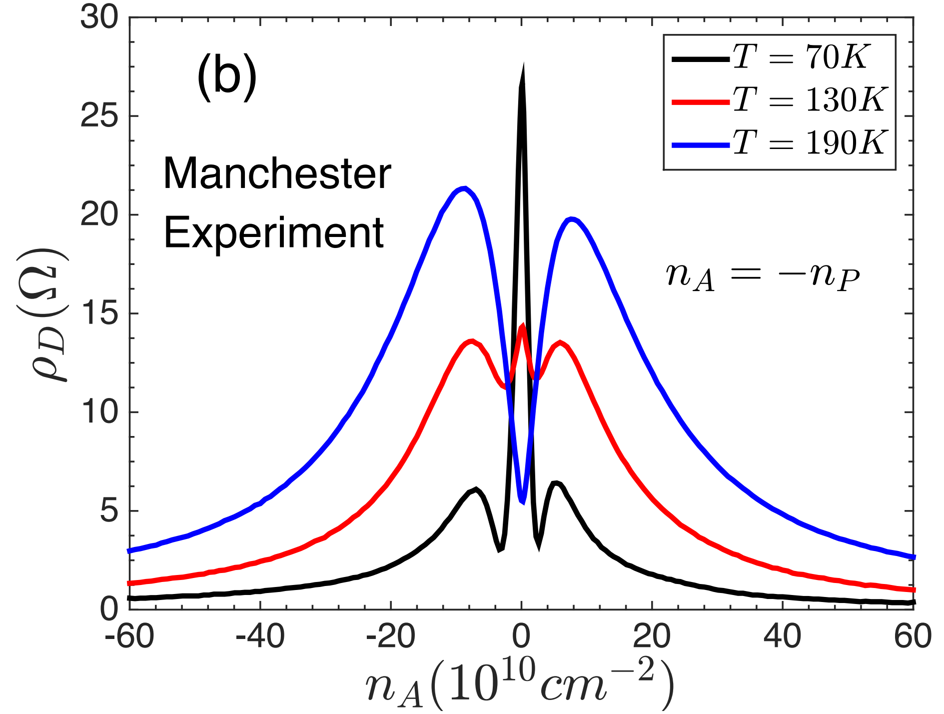
- Using the standard theory above, let us calculate ρ_D while varying:
  - Layer densities, n_A, n_P
  - Temperature, T
  - We consider B=0 only for this work.

Standard theory vs Experiment ($n_A = -n_P = n$)

Theory



Experiment
(Geim et al, Nat Phys 2012)

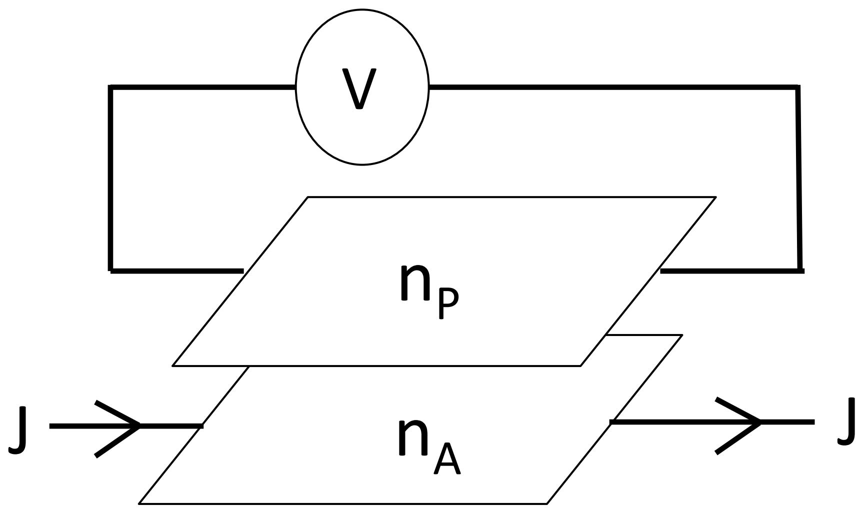


Obvious Discrepancies

- T-dependence of outer peaks!
- A central peak seen only in experiment

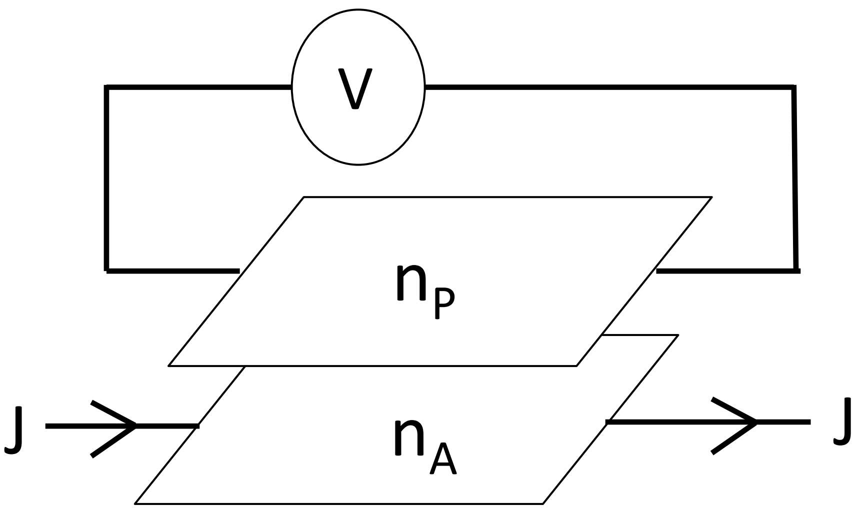
Why is there a discrepancy?

2D Materials are highly susceptible to charge density fluctuations



Ideal, No impurity-induced inhomogeneity

2D Materials are highly susceptible to charge density fluctuations



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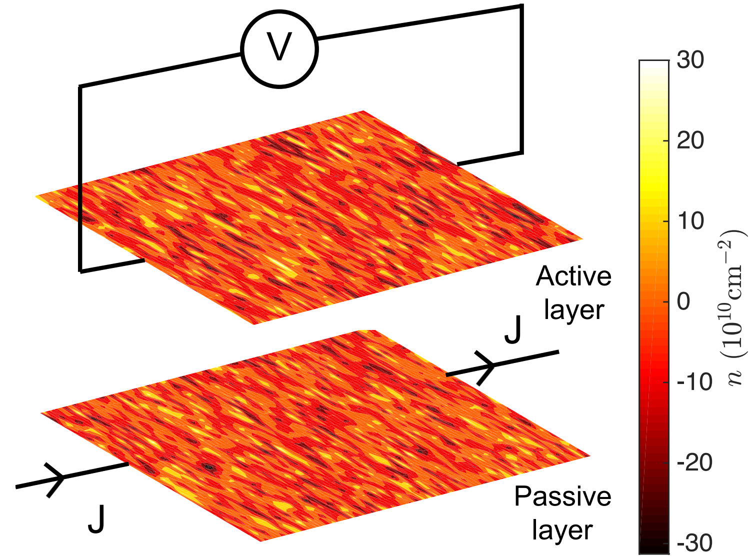
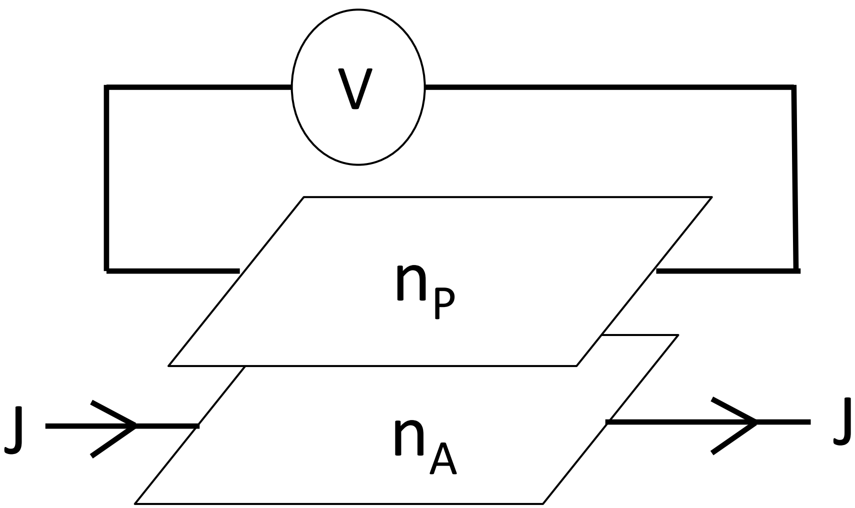


Realistic, Inhomogeneity from puddles/impurities

2D Materials are highly susceptible to charge density fluctuations



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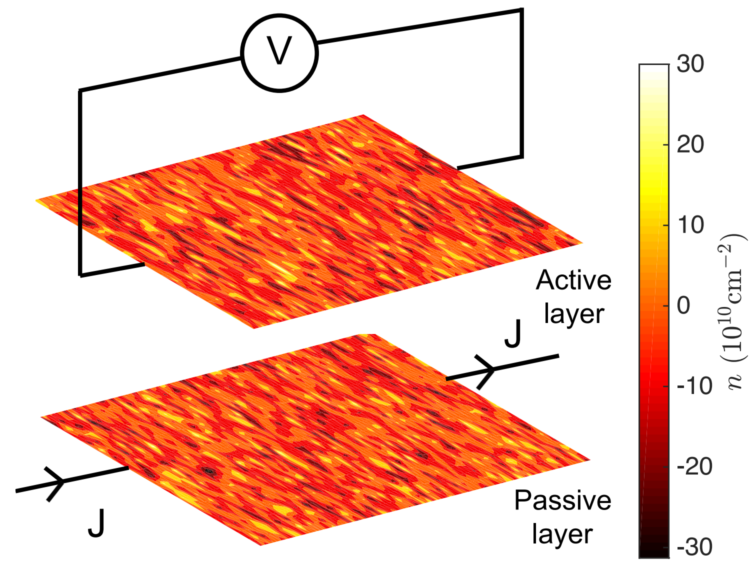


Realistic, Inhomogeneity from puddles/impurities

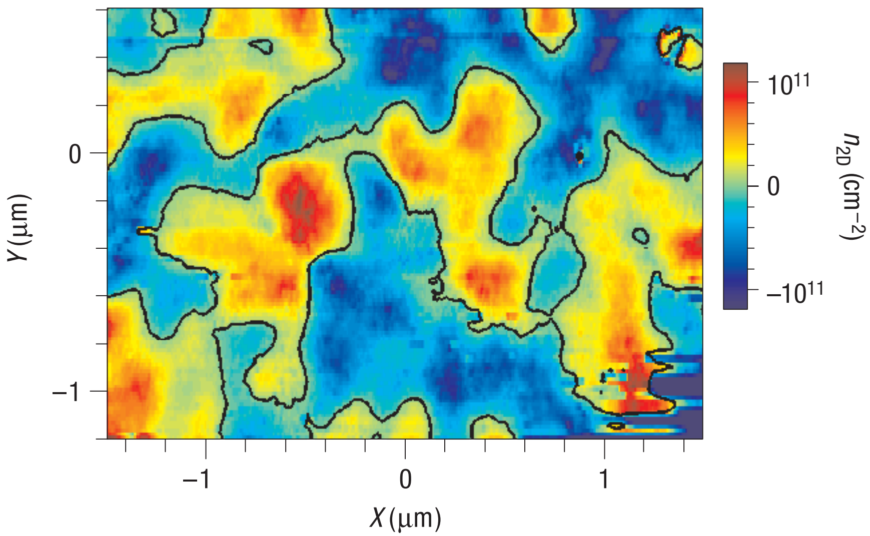
What is the effect of puddles on drag resistivity?

What about interlayer correlations between puddes?

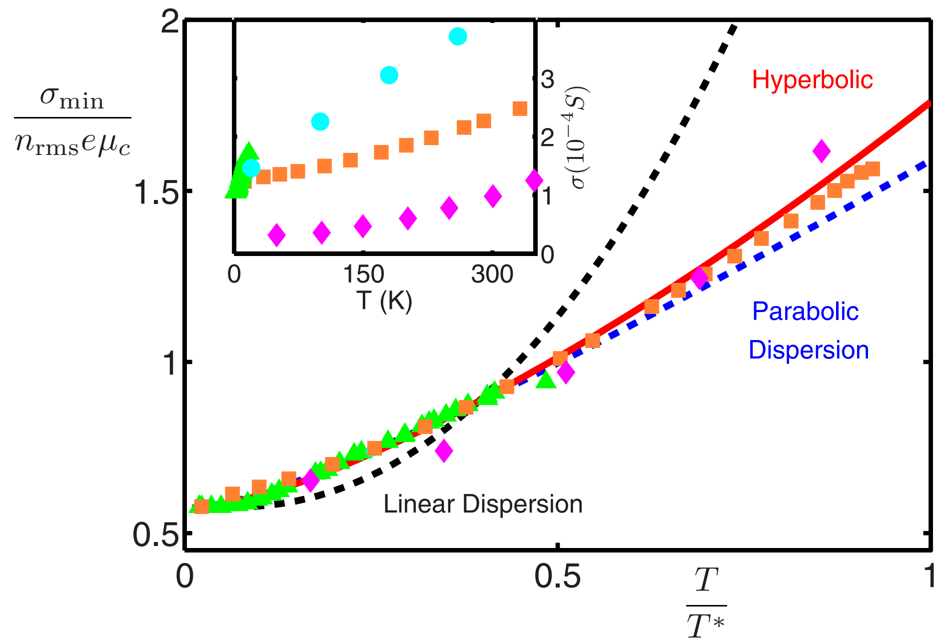
There is no established way of calculating this!

Solution: Effective Medium Theory (EMT)

EMT effectively models charge transport in graphene at the neutrality point



Puddles in monolayer graphene
(J Martin, Nat Phys 2008).

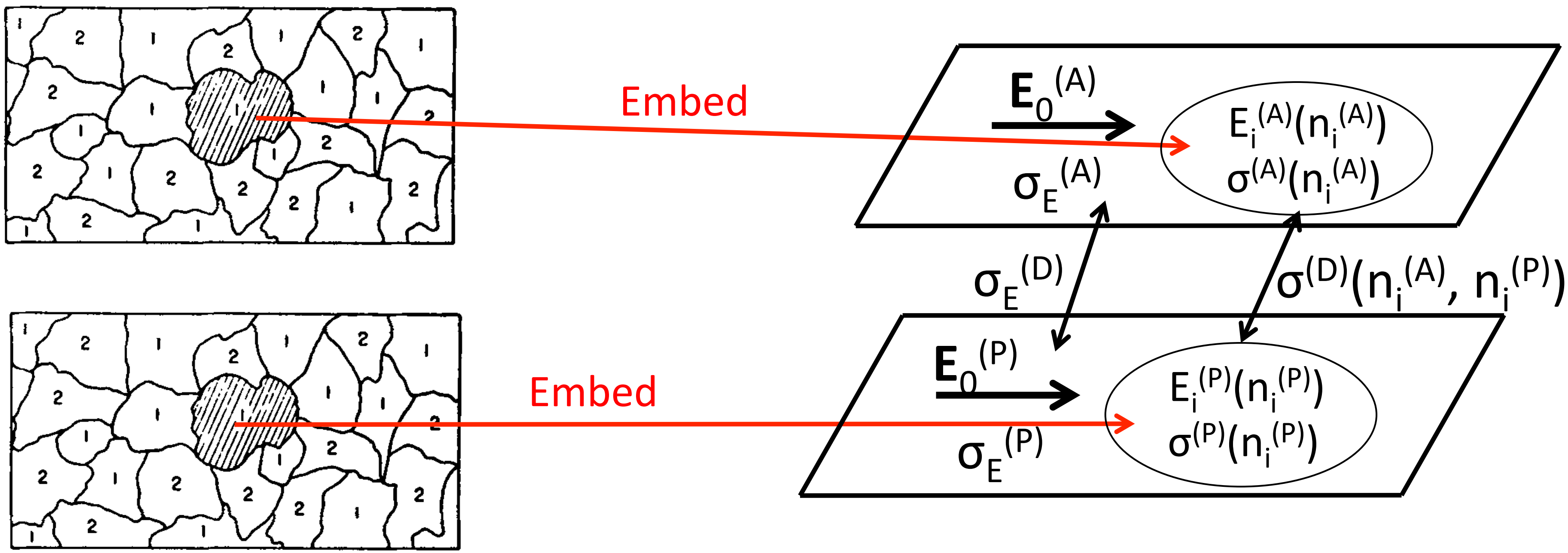


EMT applied to graphene conductivity
Adam & Stiles, Phys Rev B (2010)

We derive an effective medium theory for the Coulomb drag problem.

Brief Derivation of Drag EMT

1. Take a pair of patches. Embed them into a pair of effective media.



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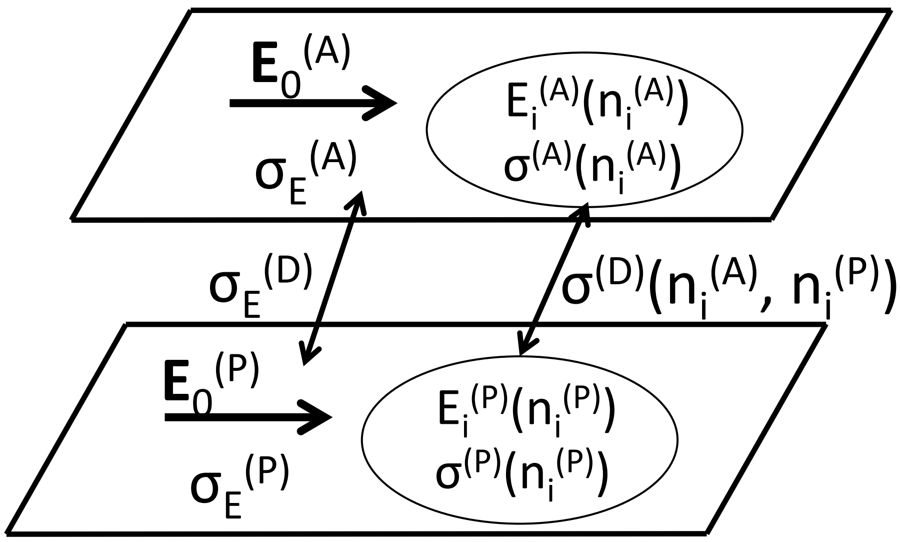

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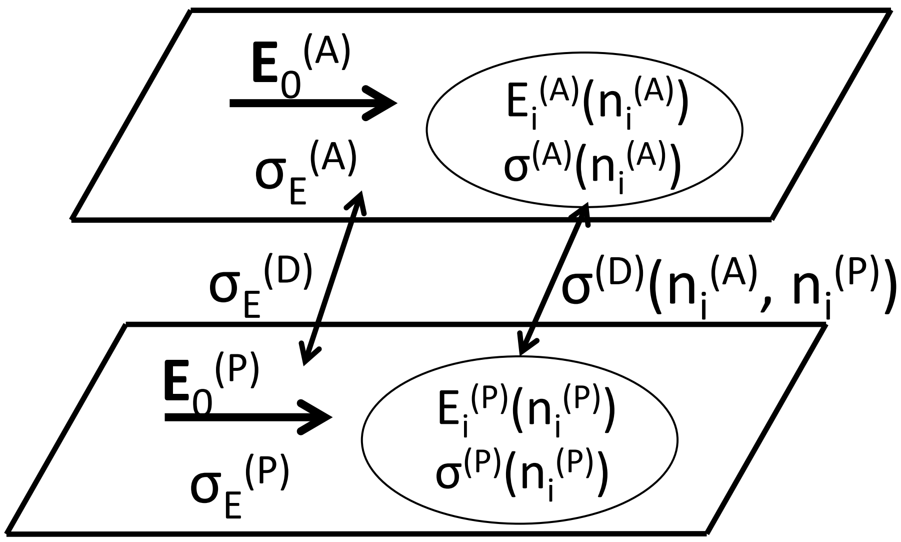
Brief Derivation of Drag EMT

1. Take a pair of patches. Embed them into a pair of effective media.

2. Solve for E-fields inside each patch $E_i^{(A)}$ and $E_i^{(P)}$.



Brief Derivation of Drag EMT

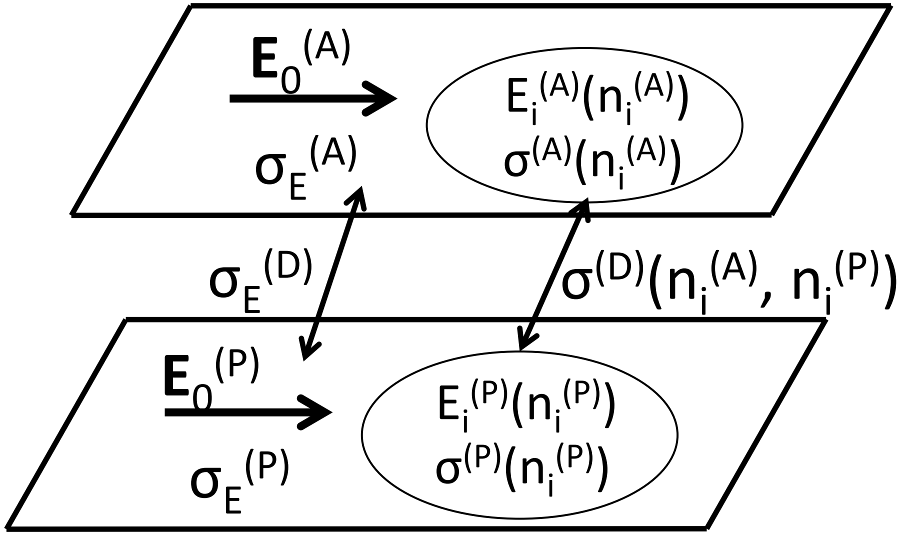
1. Take a pair of patches. Embed them into a pair of effective media.

2. Solve for E-fields inside each patch $E_i^{(A)}$ and $E_i^{(P)}$.

3. Require that average intra-patch field equals the effective medium field.

$$\sum_i P_i(n_i^{(A)}, n_i^{(P)}) E_i^{(A)} = E_0^{(A)},$$

$$\sum_i P_i(n_i^{(A)}, n_i^{(P)}) E_i^{(P)} = E_0^{(P)}$$



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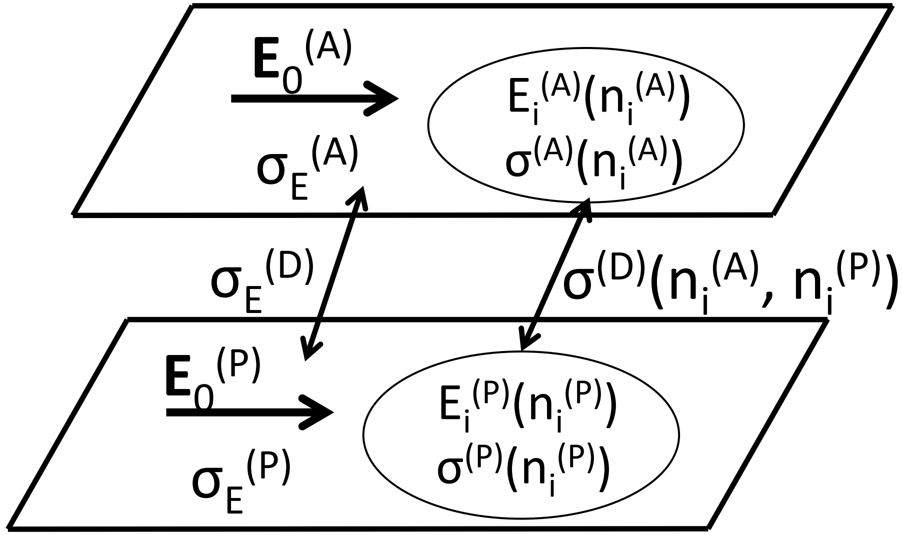

4. This yields three **EMT equations whose solutions are the effective conductivities.**

$$\int_{-\infty}^{\infty} dn'_{\mathrm{A}} P_{\mathrm{mono}}(n'_{\mathrm{A}}) \frac{\sigma_{\mathrm{A}}(n'_{\mathrm{A}}) - \sigma_{\mathrm{A}}^{\mathrm{E}}}{\sigma_{\mathrm{A}}(n'_{\mathrm{A}}) + \sigma_{\mathrm{A}}^{\mathrm{E}}} = 0,$$

$$\int_{-\infty}^{\infty} dn'_{\mathrm{P}} P_{\mathrm{mono}}(n'_{\mathrm{P}}) \frac{\sigma_{\mathrm{P}}(n'_{\mathrm{P}}) - \sigma_{\mathrm{P}}^{\mathrm{E}}}{\sigma_{\mathrm{P}}(n'_{\mathrm{P}}) + \sigma_{\mathrm{P}}^{\mathrm{E}}} = 0,$$

$$\sigma_{\mathrm{D}}^{\mathrm{E}} = \sigma_{\mathrm{A}}^{\mathrm{E}} \frac{\int_{-\infty}^{\infty} dn'_{\mathrm{A}} \int_{-\infty}^{\infty} dn'_{\mathrm{P}} P_{\mathrm{bi}}(n'_{\mathrm{A}}, n'_{\mathrm{P}}, \eta) \cdot \left[\frac{\sigma_{\mathrm{D}}(n'_{\mathrm{A}}, n'_{\mathrm{P}})}{(\sigma_{\mathrm{A}}^{\mathrm{E}} + \sigma_{\mathrm{A}}(n'_{\mathrm{A}}))(\sigma_{\mathrm{P}}^{\mathrm{E}} + \sigma_{\mathrm{P}}(n'_{\mathrm{P}}))}\right]}{\int_{-\infty}^{\infty} dn'_{\mathrm{A}} \int_{-\infty}^{\infty} dn'_{\mathrm{P}} P_{\mathrm{bi}}(n'_{\mathrm{A}}, n'_{\mathrm{P}}, \eta) \cdot \left[\frac{\sigma_{\mathrm{A}}(n'_{\mathrm{A}})}{(\sigma_{\mathrm{A}}^{\mathrm{E}} + \sigma_{\mathrm{A}}(n'_{\mathrm{A}}))(\sigma_{\mathrm{P}}^{\mathrm{E}} + \sigma_{\mathrm{P}}(n'_{\mathrm{P}}))}\right]},$$

New EMT derived.
For **any 2D material**!

$$\rho_{\mathrm{D}}^{\mathrm{E}} = -\frac{\sigma_{\mathrm{D}}^{\mathrm{E}}}{\sigma_{\mathrm{A}}^{\mathrm{E}} \sigma_{\mathrm{P}}^{\mathrm{E}} - (\sigma_{\mathrm{D}}^{\mathrm{E}})^2}$$

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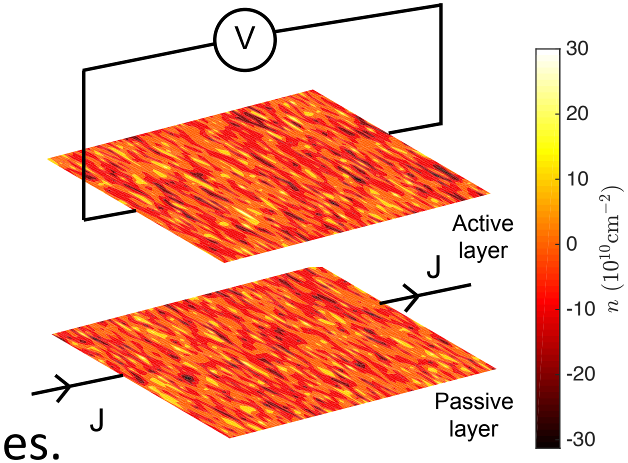
Quick Recap

Given a pair of 2D layers with non-uniform distribution of conductivities.



Solve the three EMT equations for effective conductivities.

$$\int_{-\infty}^{\infty} dn'_{\mathrm{A}} P_{\mathrm{mono}}(n'_{\mathrm{A}}) \frac{\sigma_{\mathrm{A}}(n'_{\mathrm{A}}) - \sigma_{\mathrm{A}}^{\mathrm{E}}}{\sigma_{\mathrm{A}}(n'_{\mathrm{A}}) + \sigma_{\mathrm{A}}^{\mathrm{E}}} = 0, \qquad \int_{-\infty}^{\infty} dn'_{\mathrm{P}} P_{\mathrm{mono}}(n'_{\mathrm{P}}) \frac{\sigma_{\mathrm{P}}(n'_{\mathrm{P}}) - \sigma_{\mathrm{P}}^{\mathrm{E}}}{\sigma_{\mathrm{P}}(n'_{\mathrm{P}}) + \sigma_{\mathrm{P}}^{\mathrm{E}}} = 0,$$

$$\sigma_{\mathrm{D}}^{\mathrm{E}} = \sigma_{\mathrm{A}}^{\mathrm{E}} \frac{\int_{-\infty}^{\infty} dn'_{\mathrm{A}} \int_{-\infty}^{\infty} dn'_{\mathrm{P}} P_{\mathrm{bi}}(n'_{\mathrm{A}}, n'_{\mathrm{P}}, \eta) \cdot \left[\frac{\sigma_{\mathrm{D}}(n'_{\mathrm{A}}, n'_{\mathrm{P}})}{(\sigma_{\mathrm{A}}^{\mathrm{E}} + \sigma_{\mathrm{A}}(n'_{\mathrm{A}}))(\sigma_{\mathrm{P}}^{\mathrm{E}} + \sigma_{\mathrm{P}}(n'_{\mathrm{P}}))} \right]}{\int_{-\infty}^{\infty} dn'_{\mathrm{A}} \int_{-\infty}^{\infty} dn'_{\mathrm{P}} P_{\mathrm{bi}}(n'_{\mathrm{A}}, n'_{\mathrm{P}}, \eta) \cdot \left[\frac{\sigma_{\mathrm{A}}(n'_{\mathrm{A}})}{(\sigma_{\mathrm{A}}^{\mathrm{E}} + \sigma_{\mathrm{A}}(n'_{\mathrm{A}}))(\sigma_{\mathrm{P}}^{\mathrm{E}} + \sigma_{\mathrm{P}}(n'_{\mathrm{P}}))} \right]},$$

Inputs:

1. Conductivities as function of density
2. Density distributions of the two layers.

Note that fluctuations in the layers may be correlated.

$$\rho_{\mathrm{D}}^{\mathrm{E}} = -\frac{\sigma_{\mathrm{D}}^{\mathrm{E}}}{\sigma_{\mathrm{A}}^{\mathrm{E}} \sigma_{\mathrm{P}}^{\mathrm{E}} - (\sigma_{\mathrm{D}}^{\mathrm{E}})^2}$$

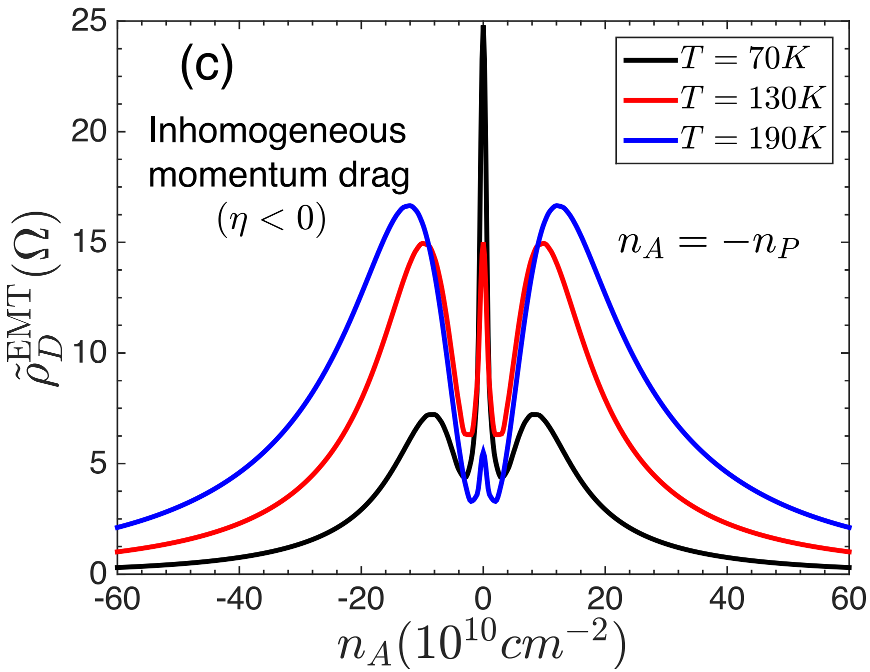
Obtain the effective drag resistivity that is measured in experiment.

Works for **any 2D material**!

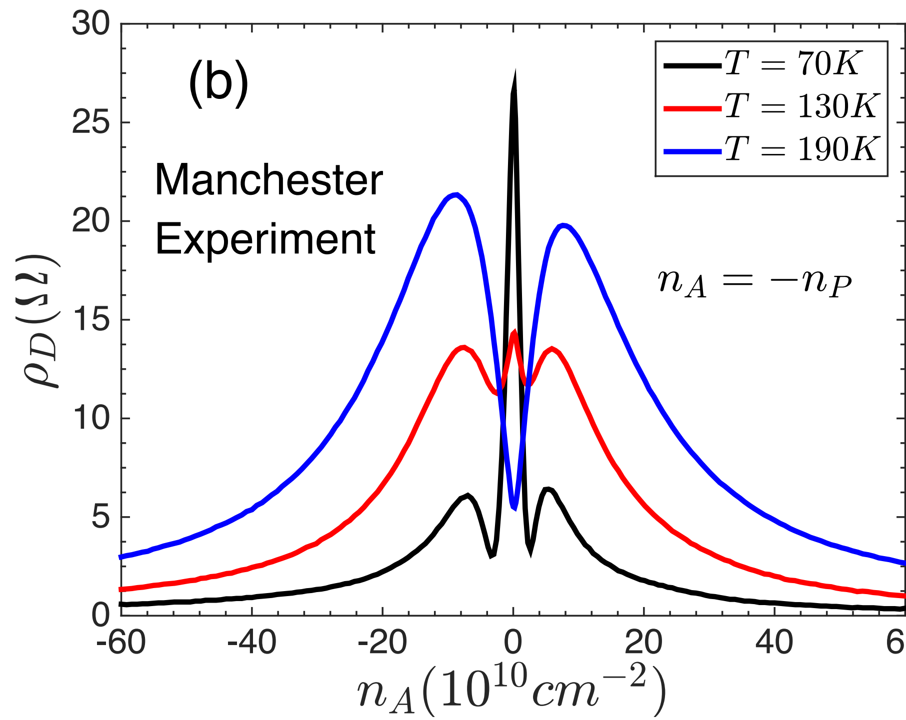
What do we get if we apply the formalism to drag in graphene?

Effective Medium theory vs Experiment ($n_A = -n_P = n$)

Effective Medium Theory



Experiment (Geim et al, Nat Phys 2012)

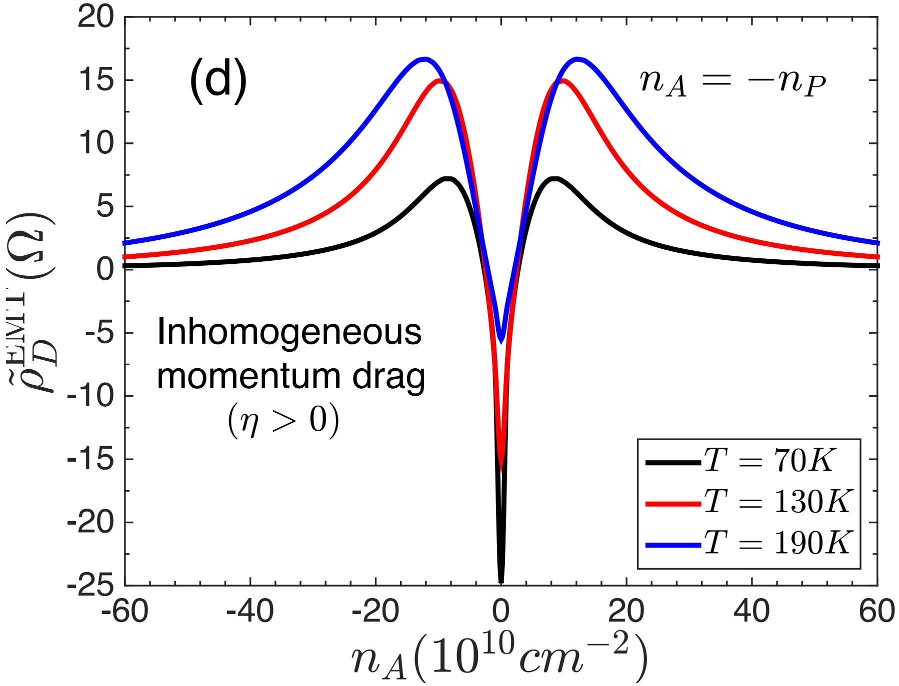


Effects of inhomogeneity/puddles

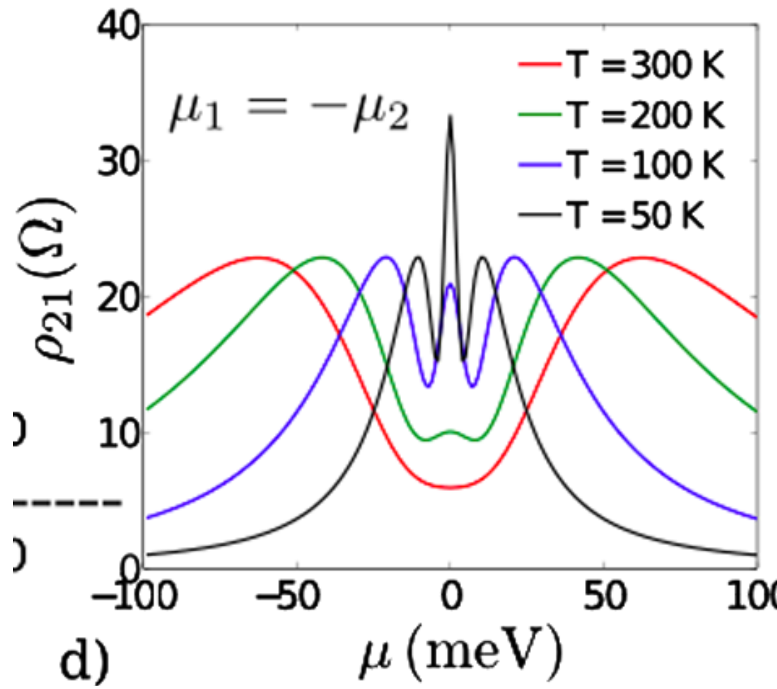
- Outer peaks increase with T
- Central peak is seen due to anti-correlated puddles.

What if the layers are correlated due to impurities?

Drag EMT



b) Song & Levitov (2012)



A strong negative peak is seen at the double Dirac point...

But this contradicts with the energy drag theory (Song & Levitov, PRL 2012).

This brings us to some unresolved problems

1. Do the layer anti-correlate due to attraction of oppositely charged puddles, or do they correlate due to common charged impurities seen by both layers?

2. Assuming they correlate/anti-correlate, which mechanism wins? Momentum or energy drag?

Both problems remain open.

Summary & Discussion

- We generalize effective medium theory to Coulomb drag for the first time. Applies to arbitrary 2D systems.
- Inhomogeneity is required to explain drag experiments, as is the case for single layer graphene transport.
- Unresolved issues at the Dirac point.
  - Correlation vs anti-correlation
  - Momentum vs energy drag
- Details in our manuscript at arXiv:1611.03089v2

Last finding: The predicted divergence in ρ_D upon exciton condensation is suppressed by puddles.

Assuming a 'dancing partner' model of exciton condensation,

$\sigma_{\mathrm{D}} = \sigma_{\mathrm{A}} = \sigma_{\mathrm{P}}$ and $\rho_{\mathrm{D}} = -\dfrac{\sigma_{\mathrm{D}}}{\sigma_{\mathrm{A}}\sigma_{\mathrm{P}} - \sigma_{\mathrm{D}}^2} \to \infty.$