Rigorous analysis of QAOA for graph clustering

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In this work, we analyse the Quantum Approximate Optimization Algorithm (QAOA) to solve a particular type of problem related to graph clustering [1-2]. We establish rigorous bounds for the efficiency of QAOA in the limit of a large problem. We treat our problem analytically which allows detailed insights to the inner workings of QAOA.

Our problem is to find communities in a dense stochastic block model, see Figure (1), with two communities. Communities are relatively dense subgraphs loosely connected with each other. Inside each community, $i = 1,2$, the link probability is denoted as d_{ii} (the probability that a randomly chosen pair of nodes has a link). Between the communities corresponding link probability is denoted as $d_{12} \ll d_{ii}$. We prove that parameters of QAOA can be tuned in such a way that in the large scale limit the clusters can be found in a finite number of shots on a quantum computer. Probability bounds are found in an analytical form, see Figure (1). Our method can be applied in such areas as quantum chemistry [3]. In Figure (2), we show quantum circuits and successful tests on a quantum computer.

References

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[2] H. Reittu and V. Kotovirta, Correctness of an application using QAOA, VIII Workshop-School on Quantum Computing and Information & VIII Workshop on Quantum Computing, VIII ⟨WECIQ|WCQ⟩ (2025)

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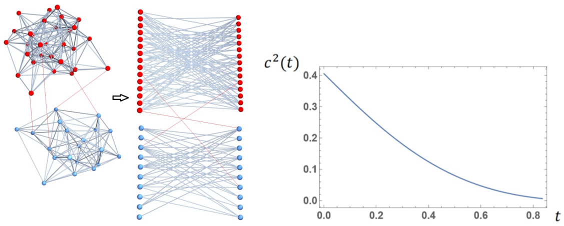
Figures



Lemma 2. $|a| \geq c(t)$, $0 < t = m/n < 1$,

$$c(t) \to \frac{2}{\pi}\int_{-1}^{1} dx \int_{-1}^{1} dy e^{-\left(\frac{\pi^2(1+t)}{4(1-t)^2}(x-\sqrt{ty})^2+2(x^2+y^2)\right)}$$

Figure 1: From top: finding communities, of sizes $2n$ and $2m$, in a graph; first split nodes into two equal sets and keep only links between the sets, making the graph bipartite. Assigning qubits to the nodes. Plot of probability of finding the red community ($c^2(t)$) as a function of the ratio of their sizes (t) in the infinite limit of the graph size. An analytical formula for the lower bound of the amplitude of the solution in QAOA method.

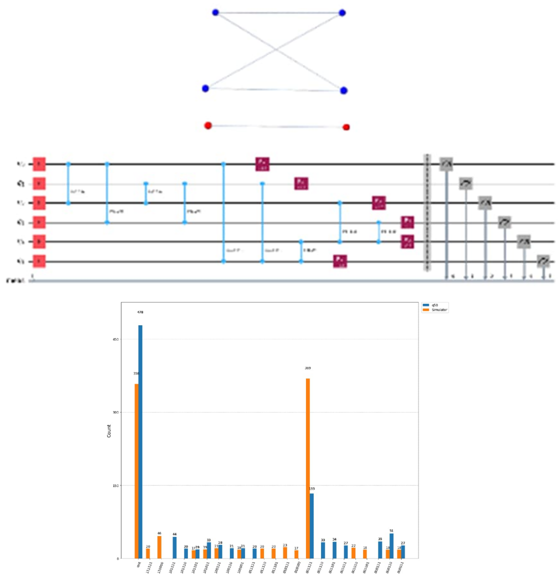


Figure 2: A test network and quantum circuit for QAOA. Gates are: one-qubit R_x and two-qubit controlled phase-shifts. Results: simulator vs. VTT's q50 quantum computer. Peak frequency ~ solution