# On temporal entropies, their scaling and measurement in many-body quantum dynamics.

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Abstract
Simulating the out-of-equilibrium dynamics of many-body quantum systems is considered one of the exponentially hard tasks for classical algorithms. We will review the origin of such exponential complexity in the language of tensor networks. We will show how such dynamics is encoded in the contraction of a spatio-temporal tensor network and thus the computational complexity coincide with the complexity necessary to contract such tensor network. Standard recipes for the contraction mimic the Schrodinger evolution of quantum states. The complexity of these states increases with time as a consequence of the linear growth of their entanglement entropy with time. Attempting to encode these states with simple tensor networks such as matrix product states translates into a computational complexity which increases exponentially in time resulting in the celebrated entanglement barrier which limit the simulations to short times. The opposite strategy, the attempt to encode the Heisenberg evolution of a local operator into a similarly simple matrix product operator is efficient for systems in which the operator entanglement grows mildly with time, which are conjectured to coincide with integrable systems. An agnostic spatio-temporal approach gives access to more flexible contraction strategies based on , rather than rather than states or operators, process tensors and influence functionals as simple tensor networks [1]. We will review our results about the complexity of such strategies which is dictated by the scaling of temporal entropies and generalized temporal entropies. They include the identification of an upper bound of temporal entropies in terms of the operator entanglement [2], the prediction of the scaling of generalized temporal entropies for critical dynamics from conformal field theories [3], and the design of specific quench protocols that allow to measure temporal entropies in experiments and use them as a witness of the nature of the dynamics [4].

References

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Figures

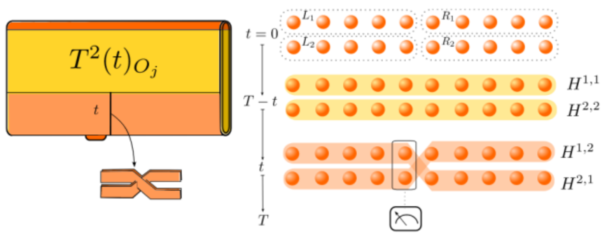


**Figure 1:** Generalized temporal entropies are encoded by the expectation value of local operators after specific double quench experiments, as described in [4]. On the left the sketch of the corresponding tensor network contraction encoding the generalized temporal purity, on the right the sketch of the idealized experiment to measure it.