# Clustering & Global Challenges

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# New Anomaly at Superconducting Instability inside a Magnetically ordered Phase

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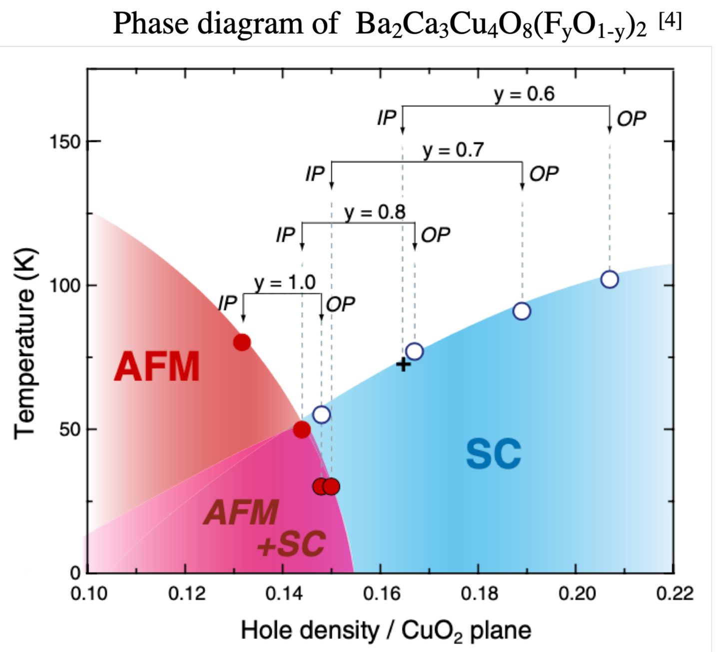
## Abstract

Superconductivity is a quantum phase where a material shows zero resistance and was discovered in 1911 by H. K. Onnes for Mercury[1]. The first successful microscopic theory of superconductivity was proposed by three prominent physicists Bardeen, Cooper, and Schrieffer in 1957[2]. The theory, later called BCS theory, successfully explained various features observed in superconductors. In the BCS theory, the superconducting instability is assumed to occur in a non-magnetic metallic phase. However, the superconducting instability can occur also inside a magnetically ordered phase, leading to the coexistence of the two ordered phases. This possibility is discussed in cuprate superconductors[3][4], heavy electron systems[5][6], and iron-based superconductors[7][8]. Since superconductivity generally competes with magnetism, it is still under debate: do the two ordered phases arise from the same electrons? or are they just phase-separated?.

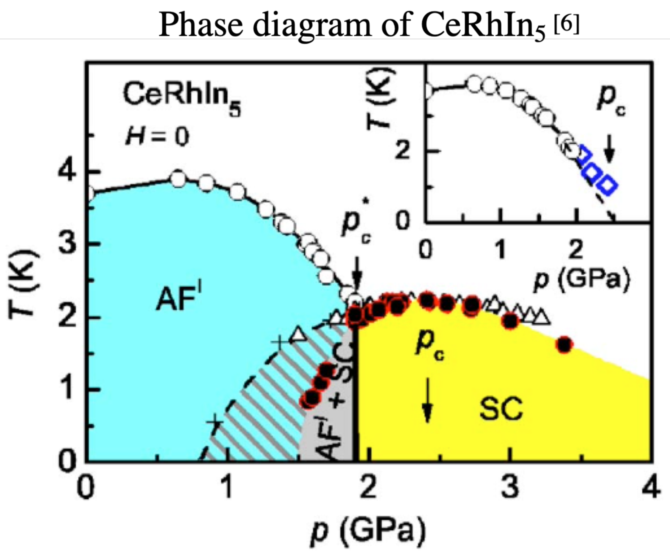
Aiming to get some insights on the above issue, we investigate the longitudinal spin susceptibility inside a magnetic phase where superconducting instability occurs via a continues phase transition and the coexistence of superconductivity and magnetism is stabilized at low temperature. What we newly find is that the spin susceptibility exhibits a jump at the superconducting transition temperature. The origin of the jump traces back to the breaking of spin rotational symmetry in the magnetically ordered phase and thus the obtained jump can be a general feature. Our finding can be utilized as a thermodynamic probe to test that the two ordered phases arise from the same electrons.

## Superconducutivity inside a magnetically ordered phase

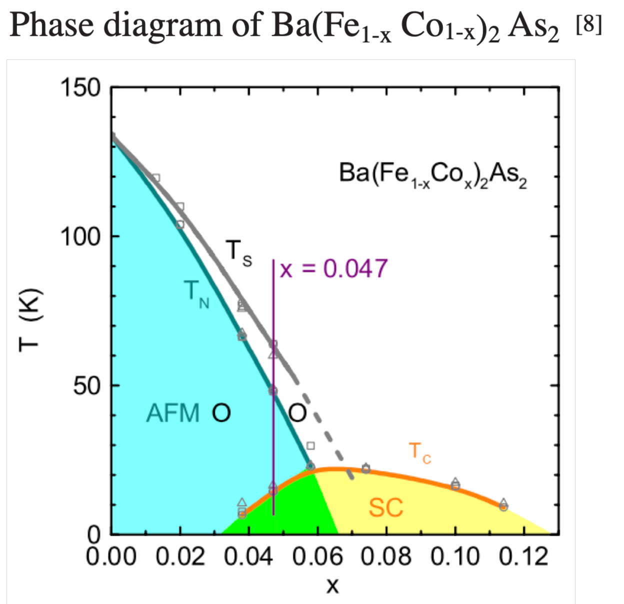
Phase diagram of $Ba_2Ca_3Cu_4O_8(F_yO_{1-y})_2$ [4]




Phase diagram of $CeRhIn_5$ [6]



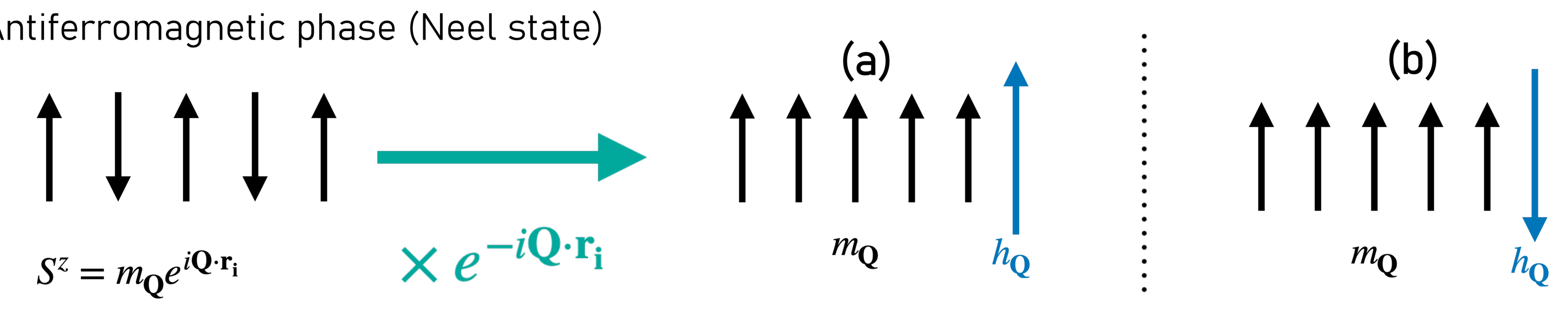

Phase diagram of $Ba(Fe_{1-x}Co_{1-x})_2As_2$ [8]




Coexistence of magnetism and superconductivity:
**Do the two ordered phases arise from the same electrons ? OR are they just phase-separated?**

## What is the point in the magnetically ordered phase?

Antiferromagnetic phase (Neel state)




$S^z = m_Q e^{i\mathbf{Q}\cdot\mathbf{r}_i}$

$\mathbf{Q} = (\pi, \pi)$

External magnetic field

Response should be different between (a) and (b)

Additional linear term in $h$ emerges

Fixed density: $\chi_n(\mathbf{Q})$

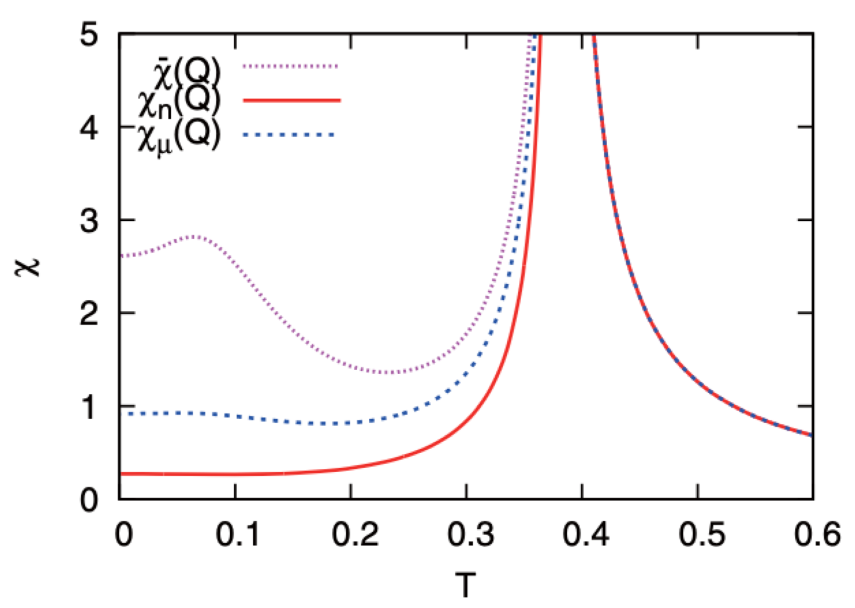
$\mu(T,h) - \mu(T,0) \approx \mu' h$

Fixed chemical potential: $\chi_\mu(\mathbf{Q})$

$n(T,h) - n(T,0) \approx n' h$

The effect is neglected: $\tilde{\chi}_n(\mathbf{Q})$

Temperature dependence of the longitudinal spin susceptibility (2D Hubbard model, mean-field) [9]




## Model and Formalism

Minimal model

$$H = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + V_s \sum_{i\tau} \hat{\Delta}^\dagger_{i\tau}\hat{\Delta}_{i\tau} + V_m \sum_{i\tau} \hat{m}_i \hat{m}_{i+\tau} - \sum_i h_i \hat{m}_i \qquad \tau = x, y$$

Superconducting channel — Magnetic channel — External field

Self-consistency equations in the mean-fields approximation:

$$n = 1 - \frac{1}{N}\sum_k{}' \left( \frac{\eta_k^+}{\lambda_k^+}\tanh\frac{\lambda_k^+}{2T} + \frac{\eta_k^-}{\lambda_k^-}\tanh\frac{\lambda_k^-}{2T} \right)$$

$$\Delta_0 = -\frac{1}{2N}\sum_k{}' (\cos k_x - \cos k_y)\left( \frac{\Delta_k}{\lambda_k^+}\tanh\frac{\lambda_k^+}{2T} + \frac{\Delta_k}{\lambda_k^-}\tanh\frac{\lambda_k^-}{2T} \right)$$

$$m = \frac{1}{2N}\sum_k{}' \frac{\bar{m}}{D_k}\left( \frac{\eta_k^+}{\lambda_k^+}\tanh\frac{\lambda_k^+}{2T} - \frac{\eta_k^-}{\lambda_k^-}\tanh\frac{\lambda_k^-}{2T} \right)$$

With :

$$\lambda_k^\pm = \sqrt{\eta_k^{\pm 2} + \Delta_k^2} \qquad \eta_k^\pm = \xi_k^+ \pm D_k$$

$$\xi_k^- = (\xi_k \pm \xi_{k+Q})/2 \qquad D_k = \sqrt{(\xi_k^-)^2 + \bar{m}^2}$$

$$\xi_k = -2\left[(t)(\cos k_x + \cos k_y) + t'\cos k_x \cos k_y + t''(\cos 2k_x + \cos 2k_y)\right] - \mu$$

The longitudinal spin susceptibility is defined as $\chi_n = \lim_{h\to 0} \frac{\partial m}{\partial h}\Big|_n$

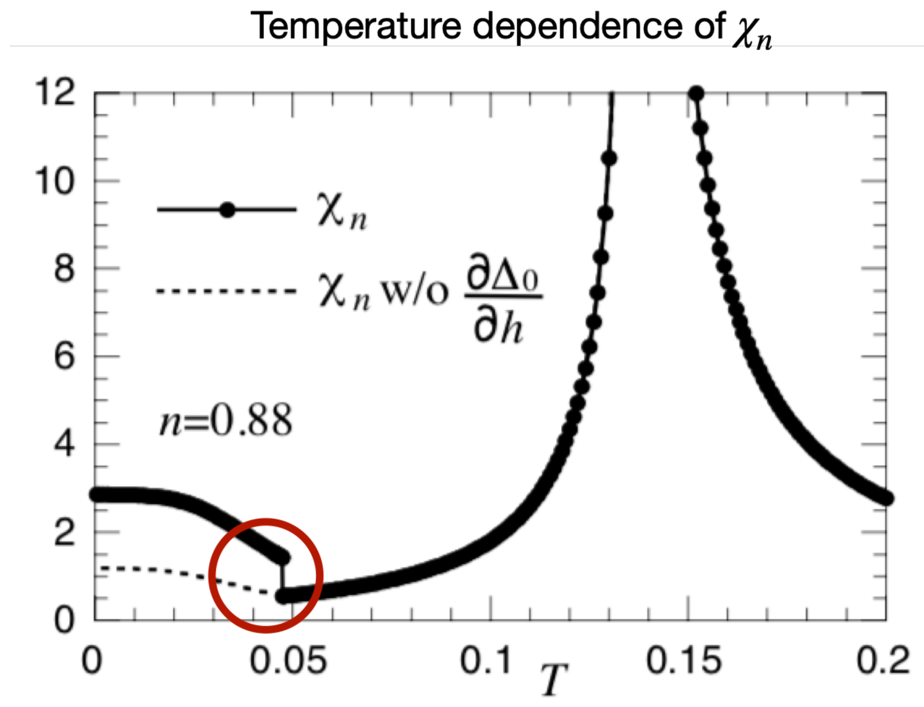
Thus in the coexistence phase :

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \frac{\partial \mu}{\partial h} \\ \frac{\partial \Delta_0}{\partial h} \\ \frac{\partial \bar{m}}{\partial h} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \chi_n \end{pmatrix} \qquad \frac{\partial \bar{m}}{\partial h} = 2V_m\chi_n + 1/2$$

The spin susceptibility has a 3 by 3 structure due to the emergence of linear terms associated with the chemical potential and the superconducting gap

## Anomaly of longitudinal spin susceptibility at Tc

Temperature dependence of $\chi_n$



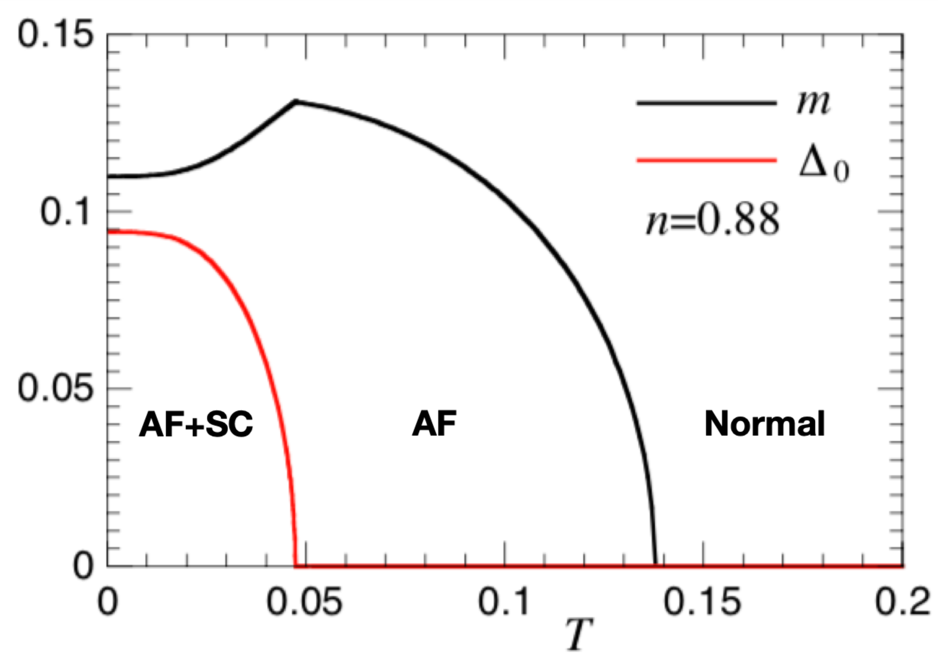

$t'/t = -0.14 \quad t/V_m = 0.8$

$t''/t = 0.07 \quad n = 0.88$

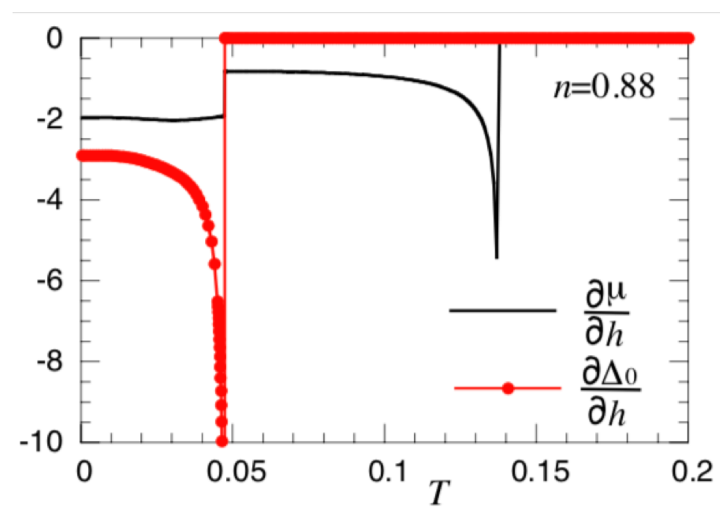
$V_s/V_m = -3/8 \ (Vs < 0)$

The spin susceptibility exhibits a jump at Tc

Temperature dependence of the order parameters




Temperature dependence of the additional terms




## Conclusions

1. We find a new anomaly of the longitudinal spin susceptibility at Tc due to the emergence of a linear term associated with the superconducting gap inside a magnetically ordered phase.
2. The anomaly reflects a general feature of the longitudinal spin susceptibility and is a manifestation of the microscopic coexistence of superconductivity and magnetism, namely the two ordered phase arise from the same electrons.

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