Fractal Analysis of distribution power grids

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Complexity of power grids arises from the geometrical arrangements and the interactions between heterogeneous components. The grid structure is meshed or radial and the networks embed multiple sources, loads interconnected by impedant lines at multiple voltage levels. Cities consume 75% of world's total primary energy and emit more than 50% of greenhouse gases. Consumption and emission are closely related to urban fabric morphology, transportation and building shape [1]. Power grids are a central element of cities' energy system. Our main motivation is hence to investigate how distribution power grids performances (voltage drops, losses) are driven by geometry and sources-loads interactions and to study how an integrated optimal design of urban fabric and distribution power networks is possible.
For that purpose, we propose to analysis both power grids and urban footprints across scales by fractal analysis [2]. We have developed some fractal and multifractal methods for characterizing power grids and built up patterns. These methods are applied to Grenoble city and IEEE distribution test cases. An approximation of the fractal dimension is obtained by a box counting method. Both the built-up patterns and the distribution power grid are covered by a number of boxes of fixed sizes. Only the non-empty boxes and the fractal dimension is obtained by identifying the power law exponent that links the number of boxes and their size. For multifractal characterization, a correlation method is used to compute the singularity spectrum of the nodal voltage values.
Fractal dimensions of built-up patterns, road network and Medium Voltage power grid of Grenoble city are first calculated. The values reflect the voids of the patterns and the influence of the bifurcations. The concordance between built-up, roads and power grid is studied and it clearly shows that higher is the building density, less concordant is the built-up pattern with the grid. Another example shows how it is possible to classify small cities in the East of France according to their fractal dimensions.
Multifractal spectrum is used to make some links between the network geometry and the voltage values. The spectrums are calculated for three IEEE test cases. This shows the diversity of the possible spectrums. This last study opens interesting issues on network planning driven by multifractal spectrum.

[1] S. M. Murshed, A. Duval, A. Koch, and P. Rode, Impact of Urban Morphology on Energy Consumption of Vertical Mobility in Asian Cities - A Comparative Analysis with 3D City Models, Urban Sci. 2019, vol. 3, no. 4.
[2] K. J. Falconer, Fractal geometry : mathematical foundations and applications, 2. ed. Chichester : Wiley, 2003.
[3] Sidqi Y., Thomas I., Frankhauser P. and, Retière N., Comparing fractal indices of electric networks to roads and buildings : The case of Grenoble (France), in Physica A : Statistical Mechanics and its Applications, Volume 531, 2019

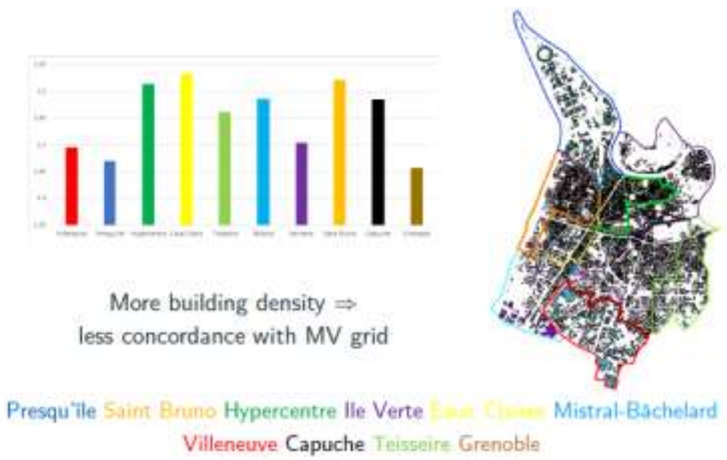

Figure 1: Concordance indices of Building vs. Grid for Grenoble city