Control of Lamb Waves by Phononic Crystal Plates

Daniel Torrent, Yabin Jin and Bahram Djafari-Rouhani

Ramón y Cajal Fellow

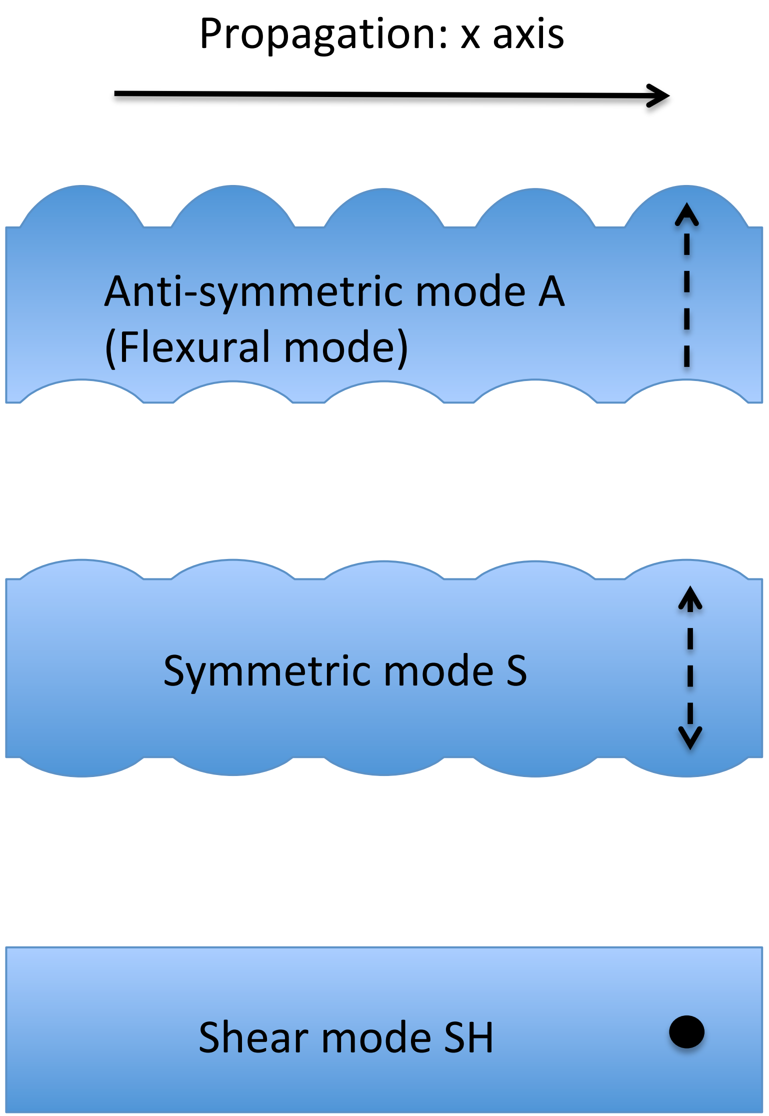
Universitat Jaume I, Castellón de la Plana (Spain)

1. Introduction
2. Refractive Devices for the A0 Mode
3. Application to the Control of the S0 Mode
4. Full Control of Lamb Waves

UNIVERSITAT JAUME I

init institute of new imaging technologies

Vibration of Phononic Crystal Plates

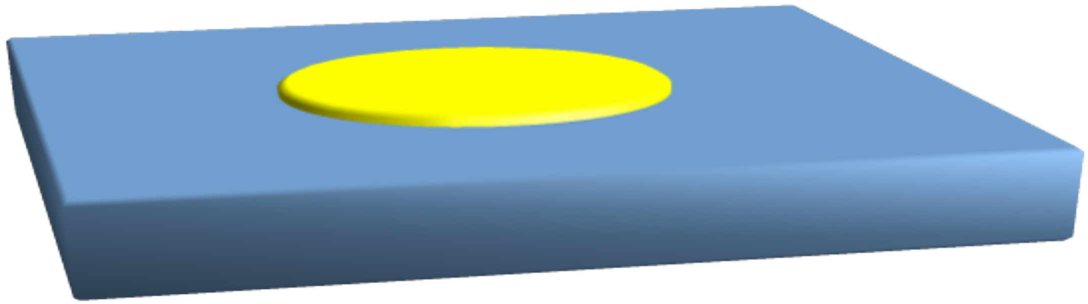


$$\vec{u} \approx u_z \vec{z} \qquad \omega \approx \frac{c_p h}{\sqrt{12}} k_A^2$$

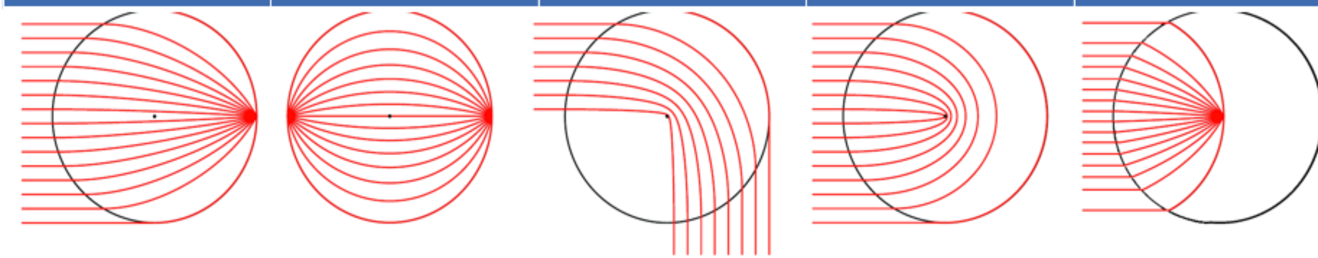
$$\vec{u} \approx u_x \vec{x} \qquad \omega \approx c_p k_s$$

$$\vec{u} \approx u_y \vec{y} \qquad \omega \approx c_{SH} k_s$$

Vibration of Phononic Crystal Plates



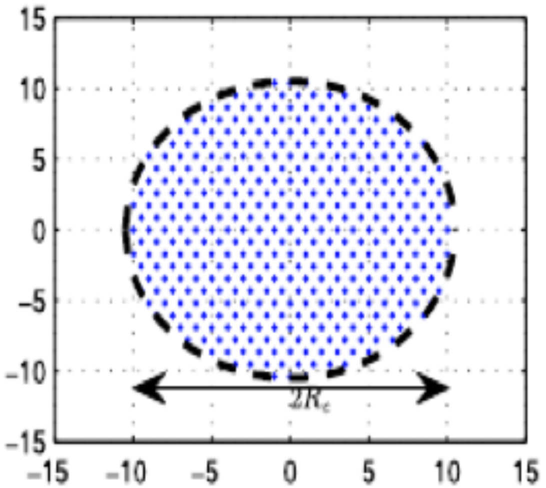
Luneburg	Maxwell	90º Rotating	Eaton	Concentrator

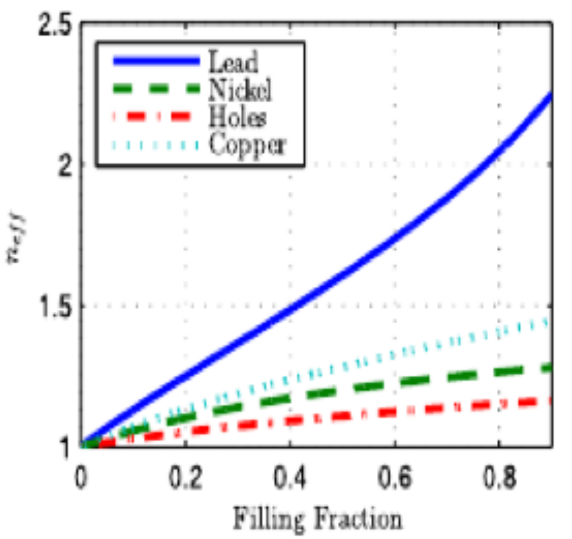


Lens Name	Refractive Index (n)
Luneburg	$n = \sqrt{2 - r^2}$
Maxwell Fish-Eye	$n = 2/(1 + r^2)$
90° Rotating	$rn^4 - 2n + r = 0$
Eaton	$n = \sqrt{2/r - 1}$
Concentrator	$n = 1/r$

Vibration of Phononic Crystal Plates

Effective Medium Theory for Flexrual Waves (A0 Mode)



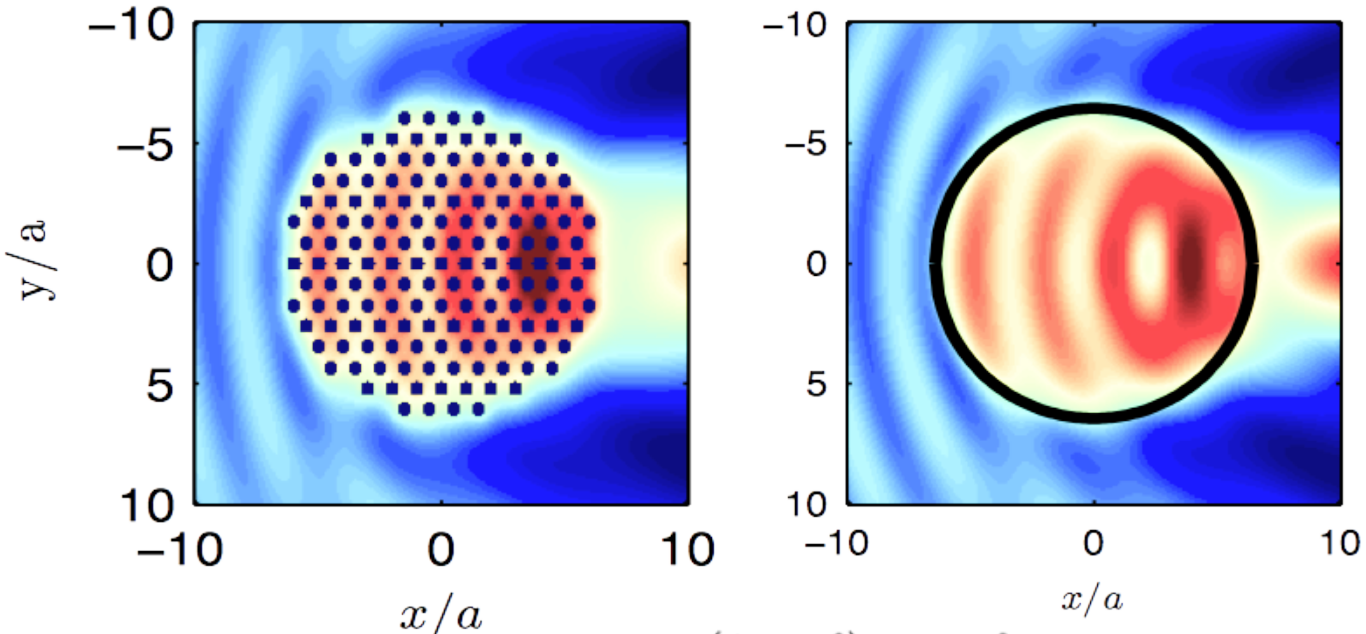


$$(D_b\nabla^4 - \rho_b h_b \omega^2)W(x,y) = 0$$

- **Graded phononic** crystals will behave like inhomogeneous materials
- The effective medium theory can be used to **locally change the filling fraction**
- The effective **refractive index will change locally** accordingly
- **Lead inclusions in Silicon** give us the best variation of the refractive index

Torrent, Pennec, Djafari-Rouhani, PRB (2014)

➘ Vibration of Phononic Crystal Plates

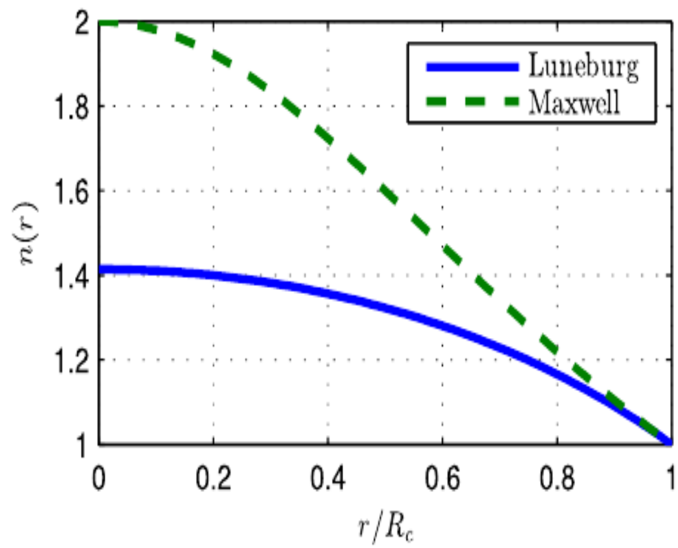


$$\rho_{eff} = (1-f)\rho_b + f\rho_a$$

$$D_{eff}(1+\nu_{eff}) = \frac{(1+\nu_b)(D_b(1-\nu_b)+D_a(1+\nu_a)) - f(1-\nu_b)(D_b(1+\nu_b)-D_a(1+\nu_a))}{D_b(1-\nu_b)+D_a(1+\nu_a) - f(D_b(1+\nu_b)-D_a(1+\nu_a)}D_b$$

$$D_{eff}(1-\nu_{eff}) = \frac{(1-\nu_b)(D_b(3+\nu_b)+D_a(1-\nu_a)) - f(3+\nu_b)(D_b(1-\nu_b)-D_a(1-\nu_a))}{D_b(3+\nu_b)+D_a(1-\nu_a) - f(D_b(1-\nu_b)-D_a(1-\nu_a))}D_b$$

Vibration of Phononic Crystal Plates

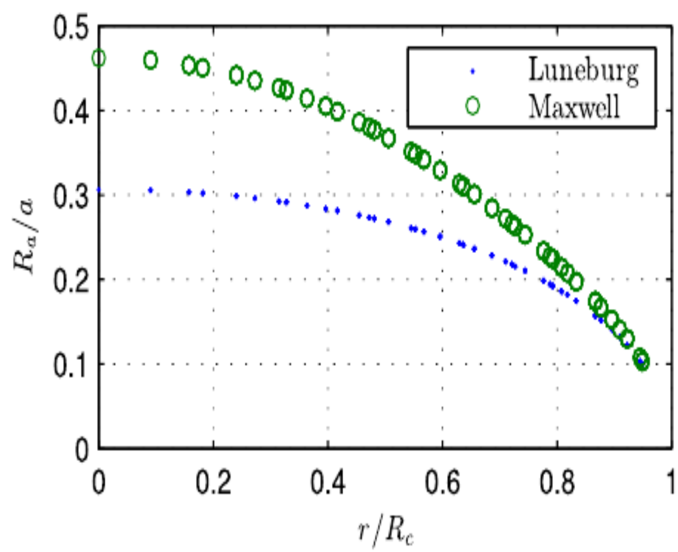


Luneburg:

$$n_L(r) = \sqrt{2 - \frac{r^2}{R_c^2}},$$

Maxwell:

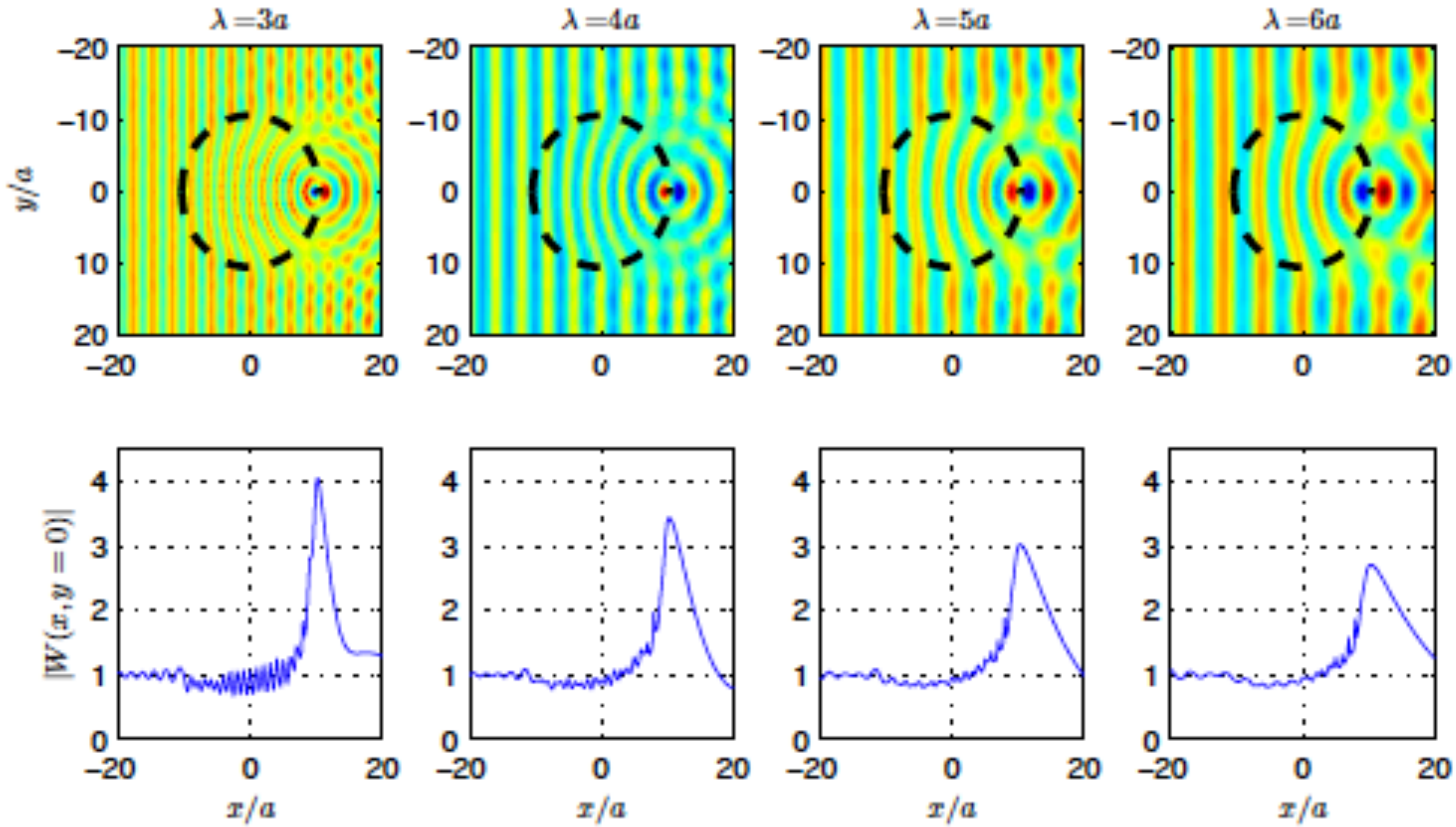
$$n_M(r) = \frac{2}{1 + (r/R_c)^2}$$



- **Luneburg and Maxwell** lens require less demanding index variations
- These two devices can be done easily
- High filling fractions are required near the center of the lens

Vibration of Phononic Crystal Plates

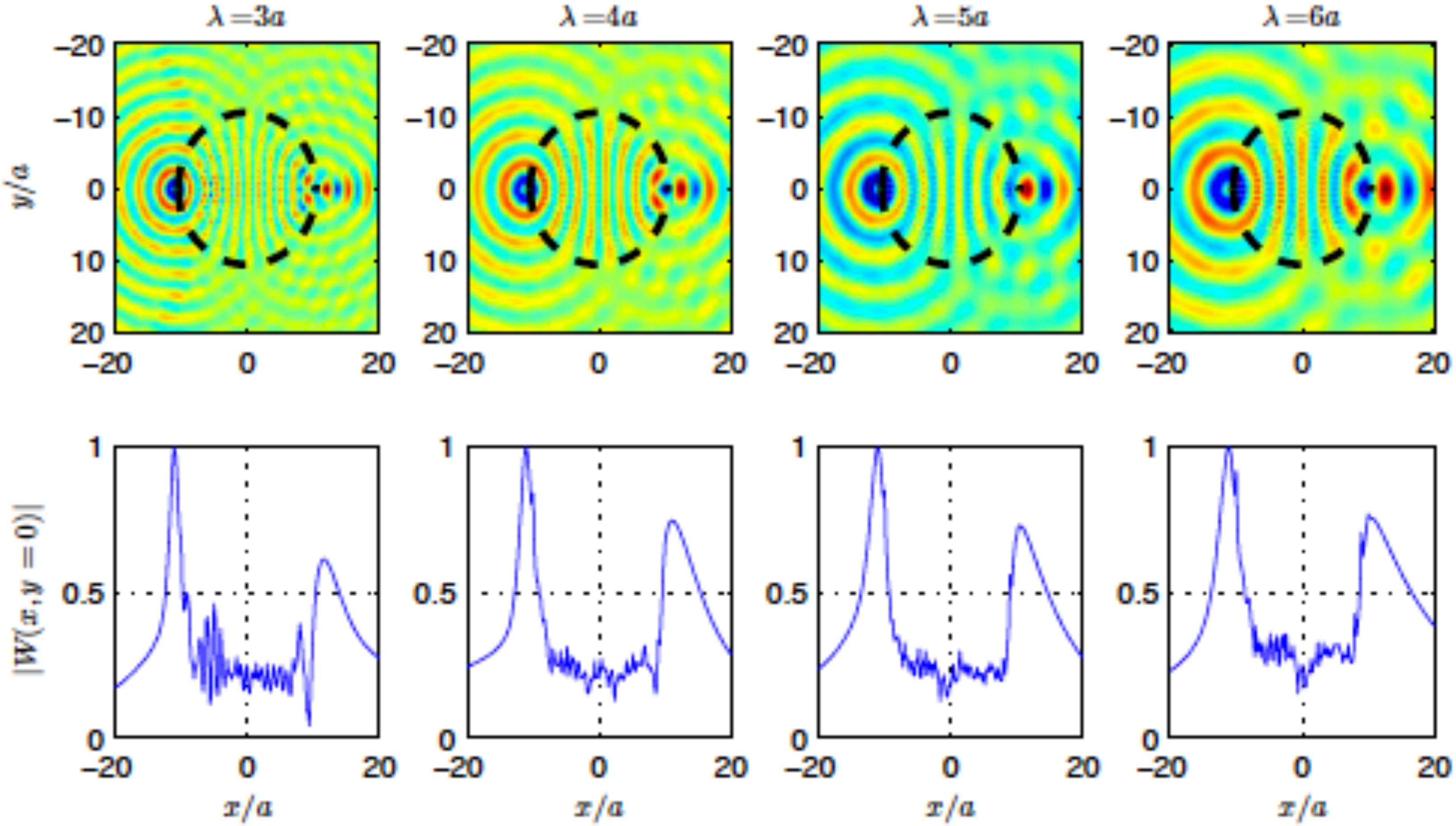
Luneburg Lens for Flexural Waves



Torrent, Pennec, Djafari-Rouhani, JAP(2014)

Vibration of Phononic Crystal Plates

Maxwell Lens for Flexural Waves



Torrent, Pennec, Djafari-Rouhani, JAP(2014)

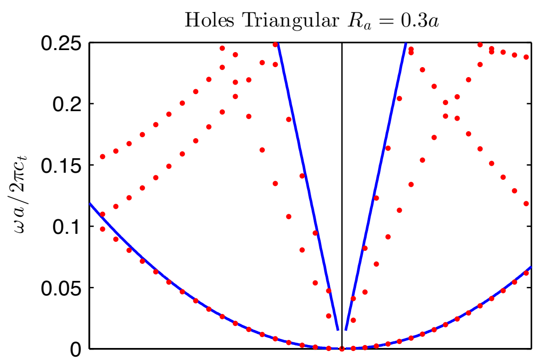
Vibration of Phononic Crystal Plates

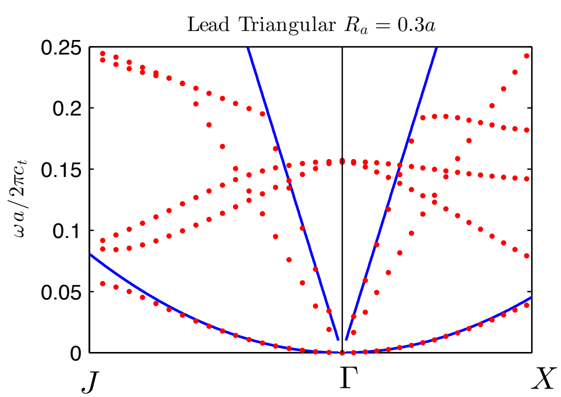
Can we also control the S_0 mode?

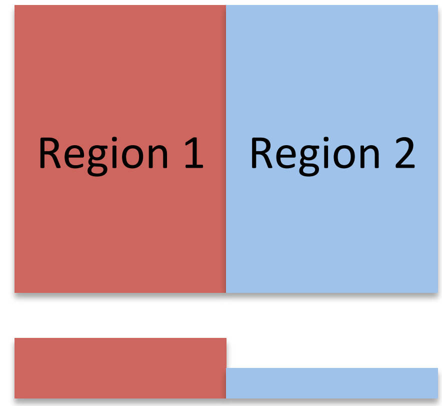
$$\omega = v_p k_S,$$

$$\omega = \frac{v_p h}{\sqrt{12}} k_A^2$$

$$v_p^2 = \frac{\overline{D^*}}{\overline{\rho^*}} \frac{E_b}{\rho_b \left(1 - \nu_b^2\right)}$$







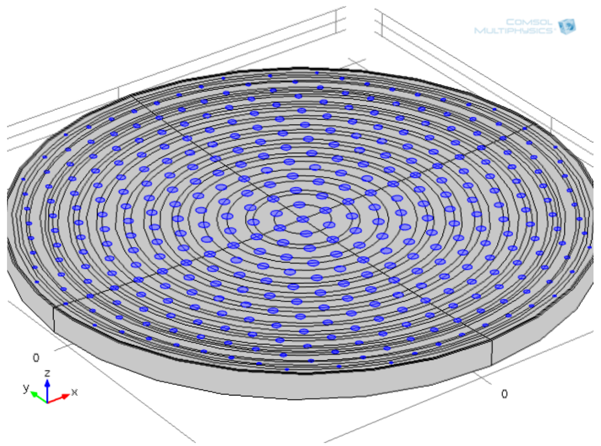
$$n_S = \frac{v_{p1}}{v_{p2}}, n_A^2 = \frac{v_{p1}}{v_{p2}} \frac{h_1}{h_2}$$

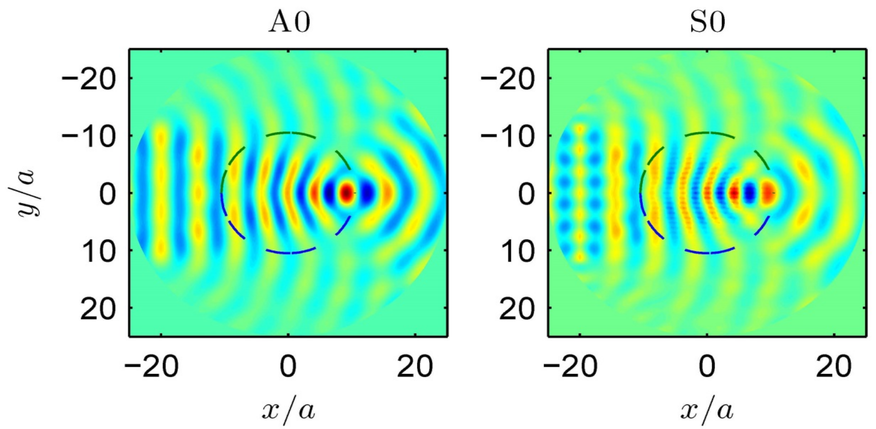
$$h_1 = h_2 n_S$$

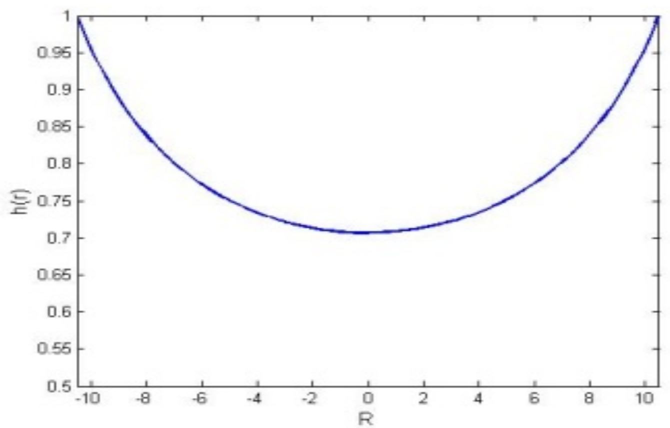
$$\boxed{n_A = n_S}$$

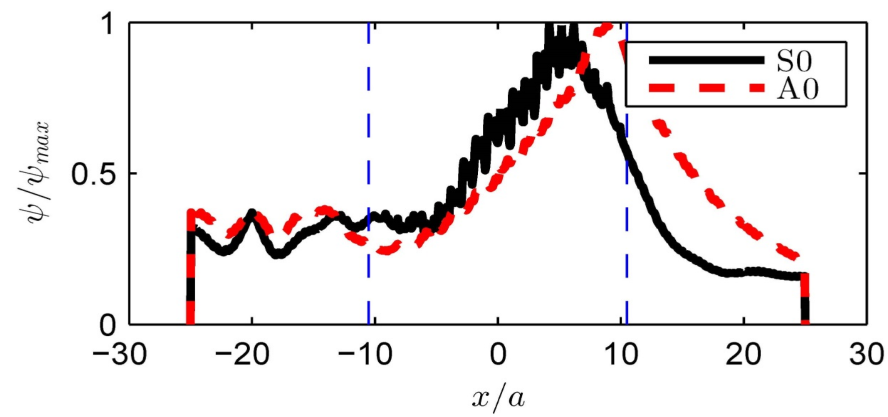
➘ Vibration of Phononic Crystal Plates

Refractive devices for both the S_0 and A_0 modes





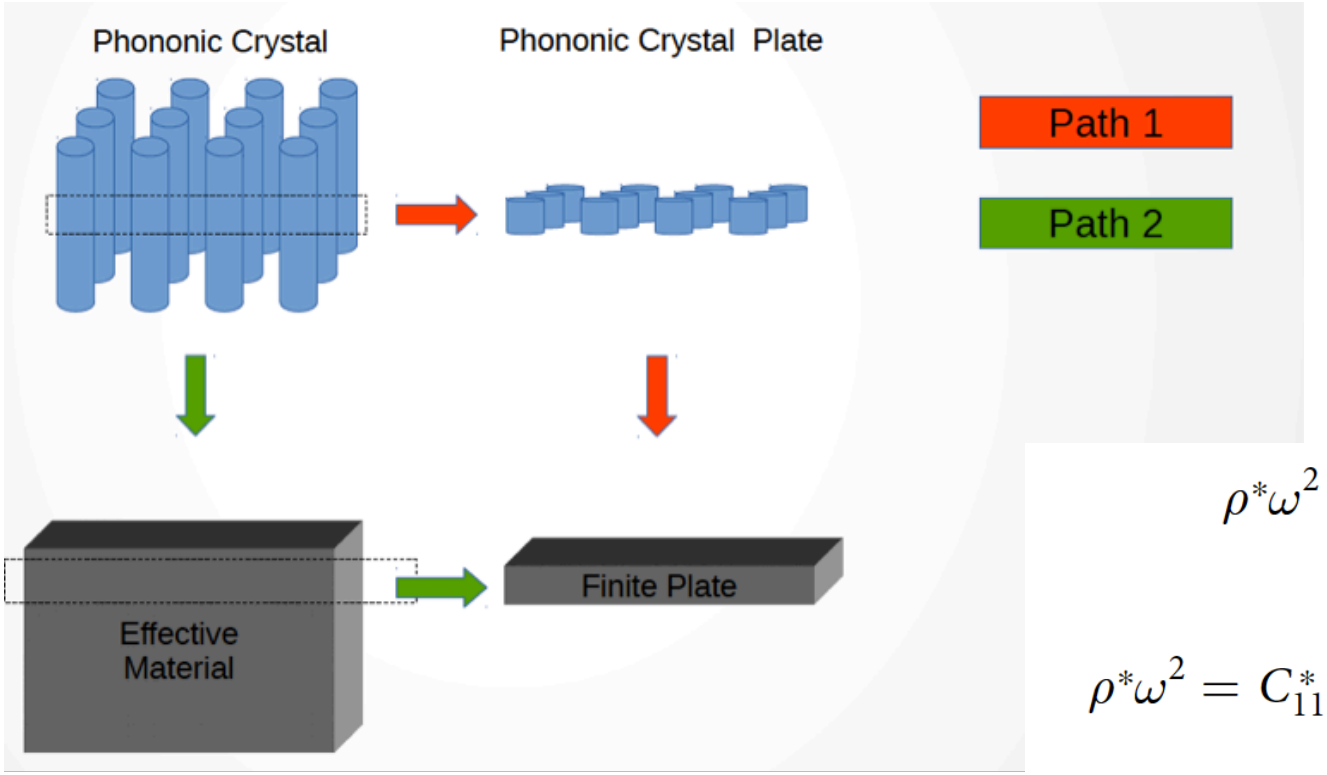




Jin, Torrent, Pennec, Pan, Djafari-Rouhani, JAP(2015)

Vibration of Phononic Crystal Plates

Can we also control the SH_0 mode?



$$\rho^*\omega^2 = C_{66}^* k_{SH}^2$$

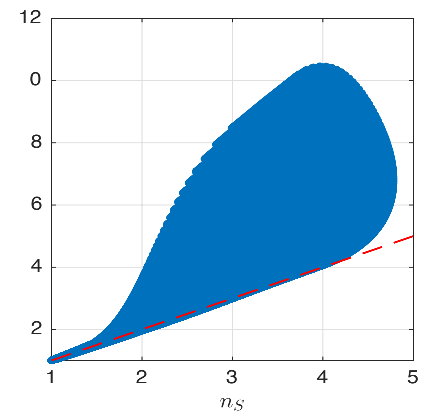
$$\rho^*\omega^2 = C_{11}^*\left(1 - \frac{C_{13}^{*2}}{C_{11}^* C_{33}^*}\right) k_S^2$$

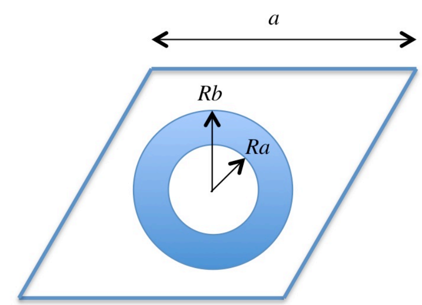
$$\rho^*\omega^2 = C_{11}^*\left(1 - \frac{C_{13}^{*2}}{C_{11}^* C_{33}^*}\right) \frac{h^2}{12} k_A^4$$

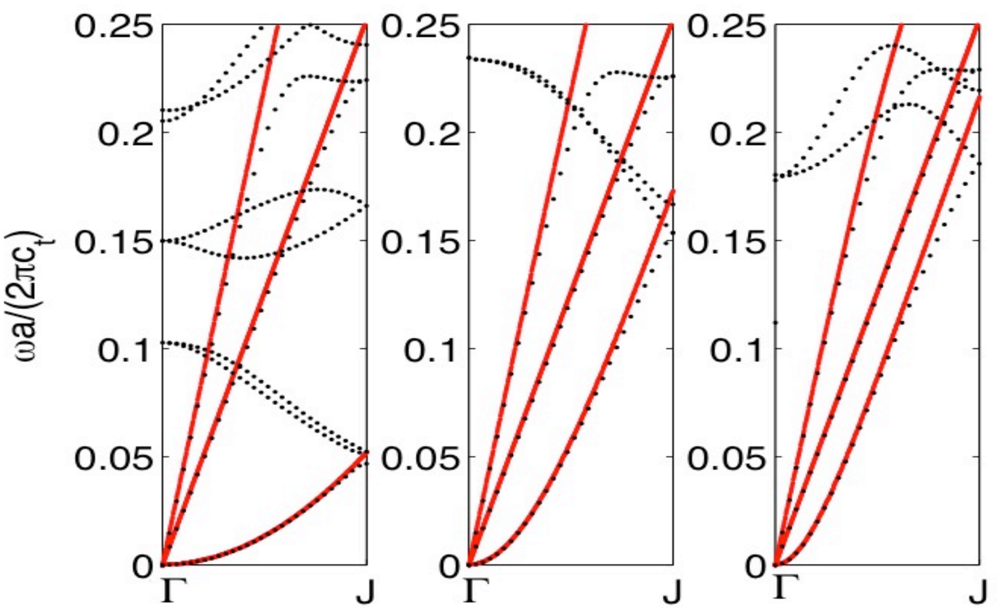
Vibration of Phononic Crystal Plates

$$\rho^* = \overline{\rho}$$

$$C^*_{IJ} = \overline{C}_{IJ} - \sum_{G,G'\neq 0} \sum_{L,\ell,m,M} C_{IL}^{-G'} G'_{L\ell} \chi_{\ell m}^{G'G} G_{mM} C_{MJ}^{G}$$







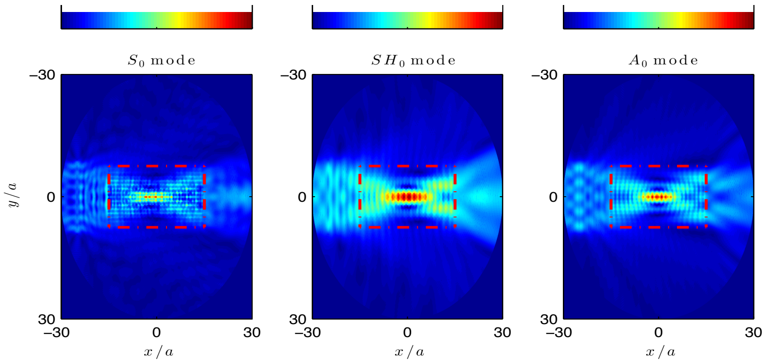
h_0, R_a, R_b

↓

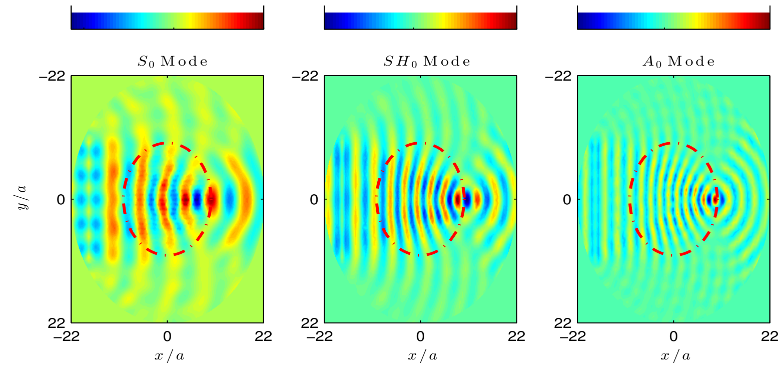
A_0, S_0, SH_0

➘ Vibration of Phononic Crystal Plates

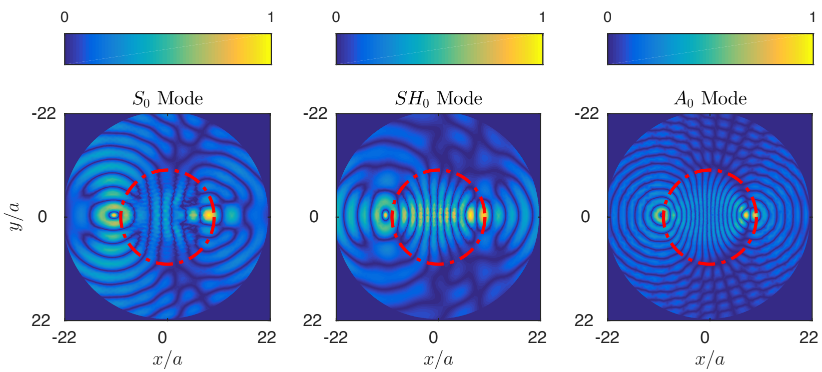
GRIN Lens



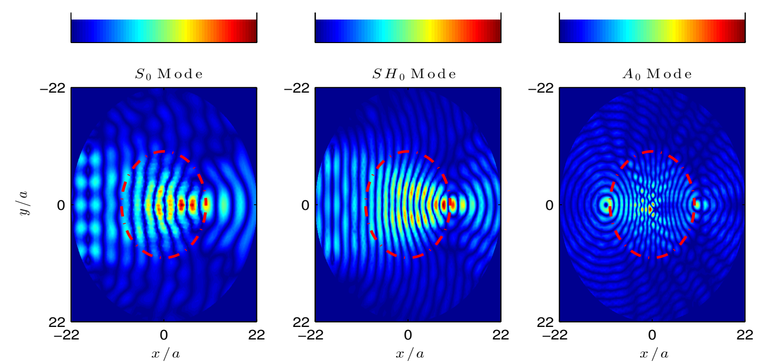
Luneburg Lens



Maxwell Lens



LN + LN + MX



Jin, Torrent, Pennec, Pan, Djafari-Rouhani, Sci. Rep(2016)

Vibration of Phononic Crystal Plates

1. Homogenization theory for Flexural Waves, PRB (09-2014)
2. Refractive devices for the A_0 mode, JAP (12-2014)
3. Refractive devices for the A_0 and the S_0 modes, JAP (06-2015)
4. Homogenization theory for Bulk Elastic Waves, PRB (11-2015)
5. Refractive devices for the $A_{0,}$ S_0 and the SH_0 modes, Sci. Rep. (04-2016)
6. Beam splitters for the $A_{0,}$ S_0 and the SH_0 modes, AIP Advances (12-2016)
7. Invisible lens for flexural waves, JPhysD. (05-2017)
8. Analysis of the robustness of topological edge states, Submitted (2018)

UNIVERSITAT JAUME I

init
institute of new imaging technologies

FEDER
Fondo Europeo de Desarrollo Regional
UNIÓN EUROPEA
"Una manera de hacer Europa"

GOBIERNO DE ESPANA
MINISTERIO DE ECONOMIA Y COMPETITIVIDAD

Office of Naval Research
USN
ONR
Science & Technology

Thank you!!