

# Nonlinear dynamics and chaos in Optomechanical nanobeams

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# Challenges in Cavity Optomechanics

## Linear Optomechanics

- Displacement detection
- Optical Spring
- Cooling & Amplification
- Two-tone drive: “Optomechanically induced transparency”
- Ground state cooling
- State transfer, pulsed operation
- Wavelength conversion
- Radiation Pressure Shot Noise
- Squeezing of Light
- Squeezing of Mechanics
- Light-Mechanics Entanglement
- Accelerometers
- Single-quadrature detection, Wigner density
- Optomechanics with an active medium
- Measure gravity or other small forces
- Mechanics-Mechanics entanglement
- Pulsed measurement
- Quantum Feedback
- Rotational Optomechanics

## Nonlinear Optomechanics

- Self-induced mechanical oscillations
- Attractor diagram?
- Synchronization of oscillations
- Chaos

OM cavities as versatile building blocks to be used for generating **reference signals**, in **neurocomputational networks**, for **chaos-based secure communications**, etc.

## Nonlinear Quantum Optomechanics

- QND Phonon number detection
- Phonon shot noise
- Photon blockade
- Optomechanical “which-way” experiment
- Nonclassical mechanical q. states
- Nonlinear OMIT
- Noncl. via Conditional Detection
- Single-photon sources
- Coupling to other two-level systems
- Optomechanical Matter-Wave Interferometry

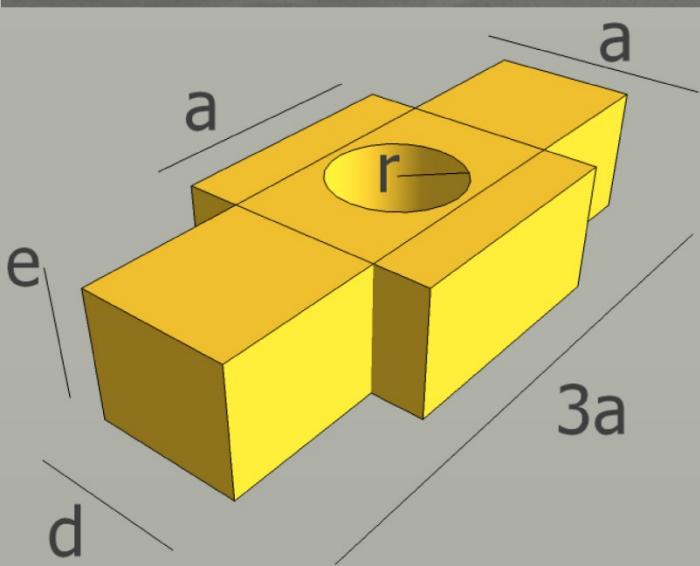
## Multimode

- Mechanical information processing
- Bandstructure in arrays
- Synchronization/patterns in arrays
- Transport & pulses in arrays

HV 10.00 kV	det ETD	mag □ 26 122 x	HFV 11.4 μm	WD 15.9 mm	spot 2.5	5 μm
Quanta FEG						

# Optomechanical cristal photonic cavities

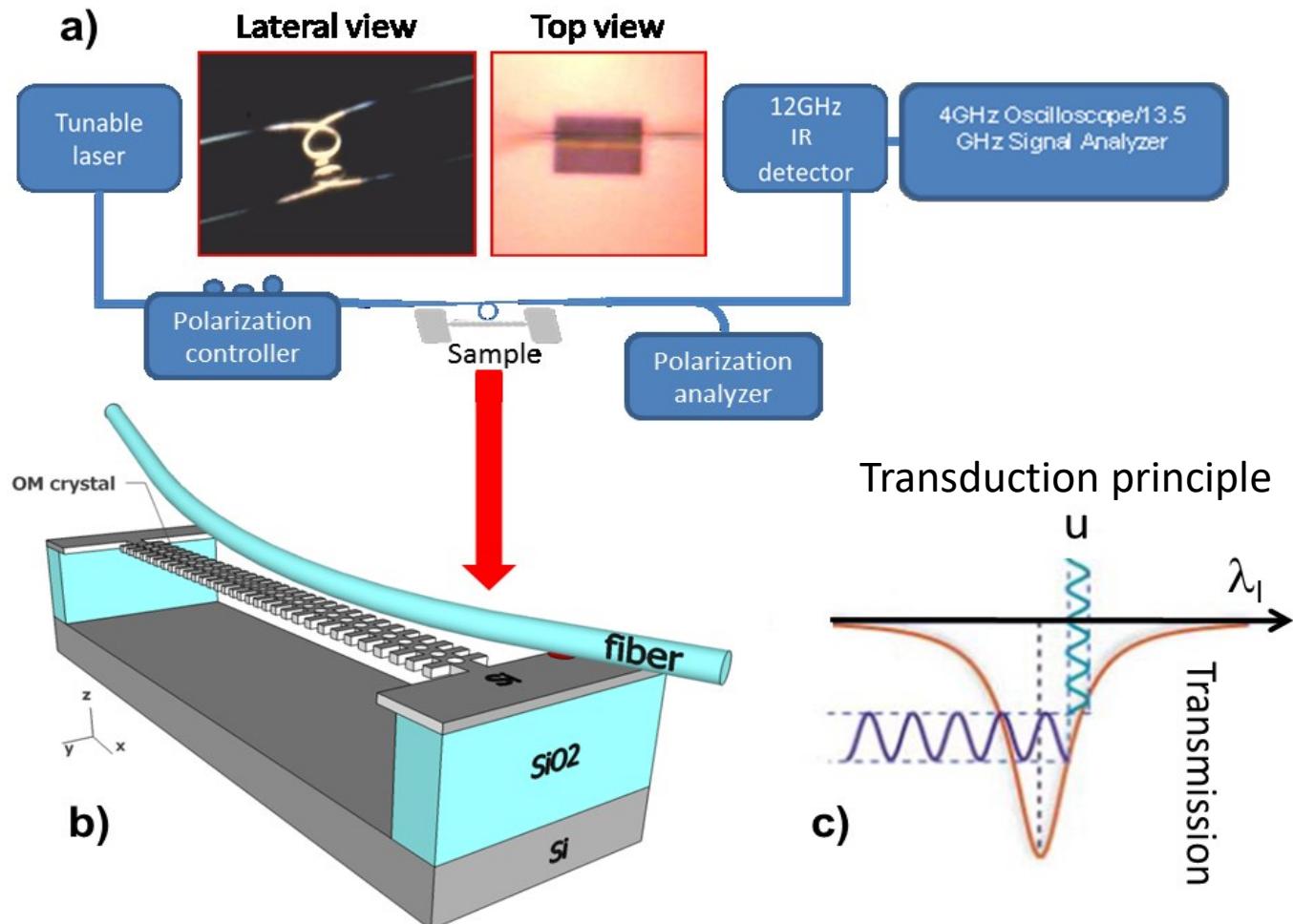
*Quadratic reduction of the pitch,  
hole radius and stub width*



J. Gomis-Bresco, D. Navarro-Urrios et al., *Nat. Comm.*, **5**, 4452 (2014)

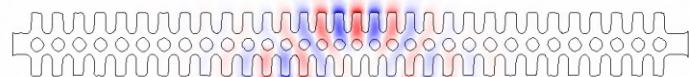
# Experimental Setup

- Standard tapered fiber set-up for characterizing optical and mechanical properties of OM devices.
- Mounted on an anti-vibration cage at **atmospheric conditions of air pressure and temperature**



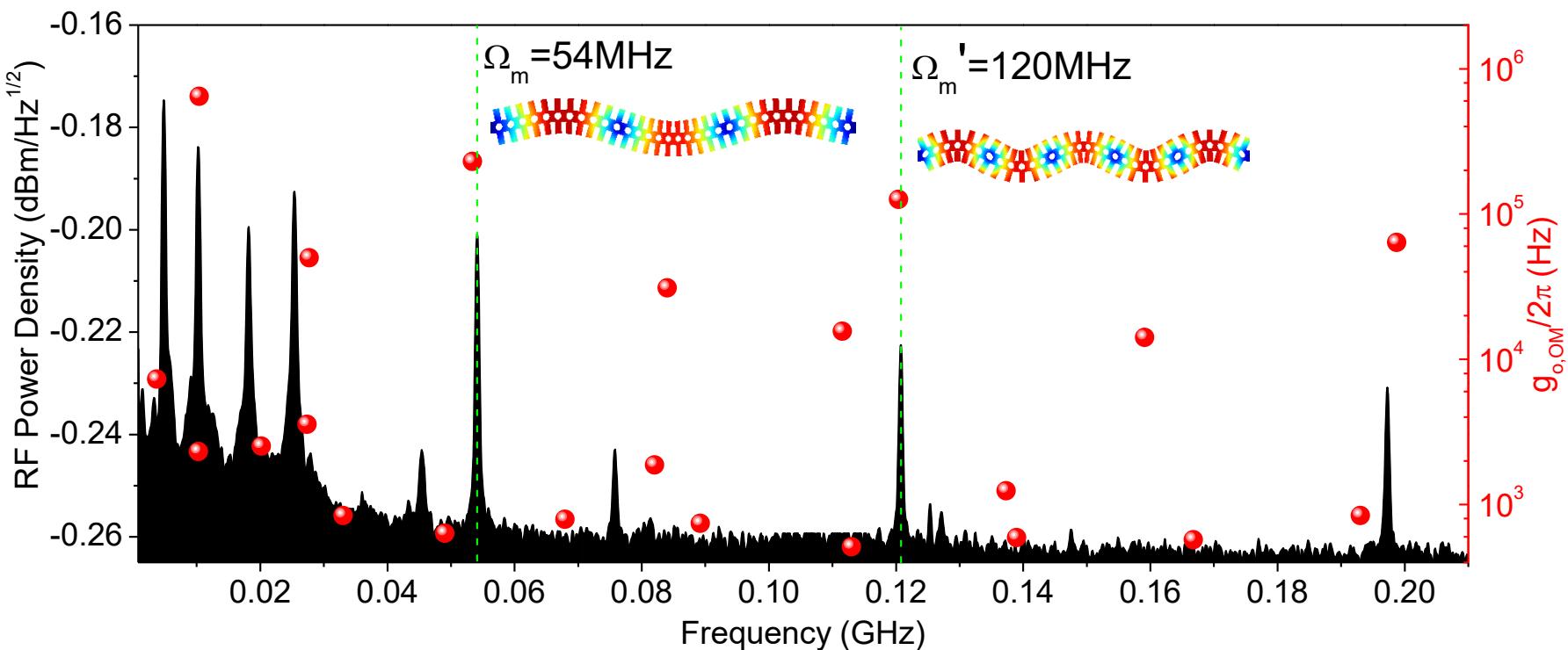
# Low frequency mechanical modes

196 THz



Side-band unresolved:  $\frac{\Omega_m}{\kappa} \sim$

In-plane, flexural modes



# Thermo-Optic/Free-Carrier-Dispersion self-pulsing

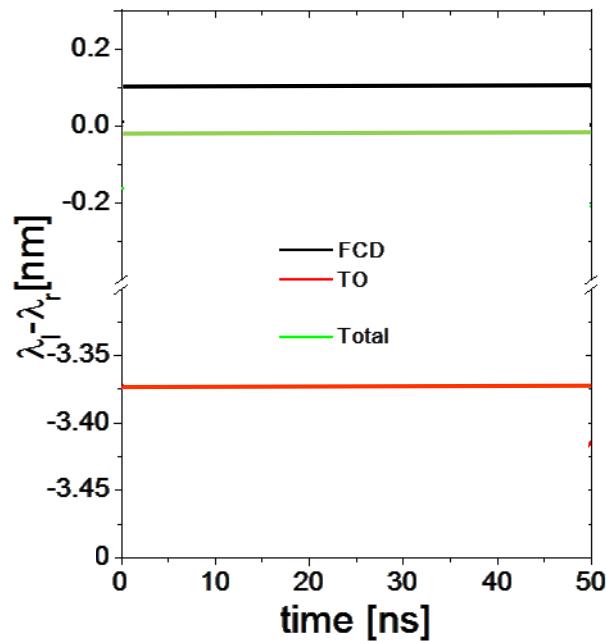
$$\lambda_r \approx \lambda_o + \frac{\partial \lambda_r}{\partial T} \Delta T - \frac{\partial \lambda_r}{\partial N} N$$

TO                    FCD

$\lambda_r$  = Resonance wavelength

N=Free carrier density

$\Delta T$ = Temperature Increase



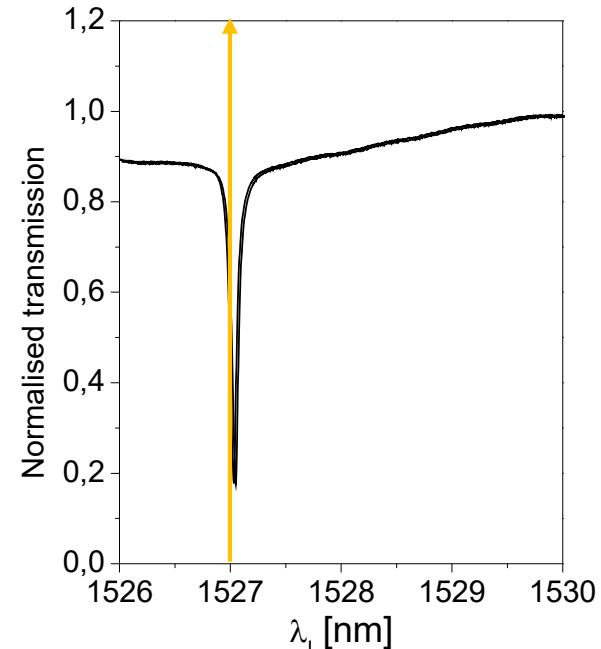
$$\frac{dN}{dt} = -\frac{1}{\tau_{FC}} N + \beta \left( \frac{hc^3}{n^2 \lambda_o V_o} \right) n_o^2;$$

$$\frac{d\Delta T}{dt} = -\frac{1}{\tau_T} \Delta T + \alpha_{FC} N n_o;$$

Number of intracavity photons

$$n_o = n_{o,max} \left( \frac{\Delta \lambda_o^2}{4(\lambda_l - \lambda_r)^2 + \Delta \lambda_o^2} \right)$$

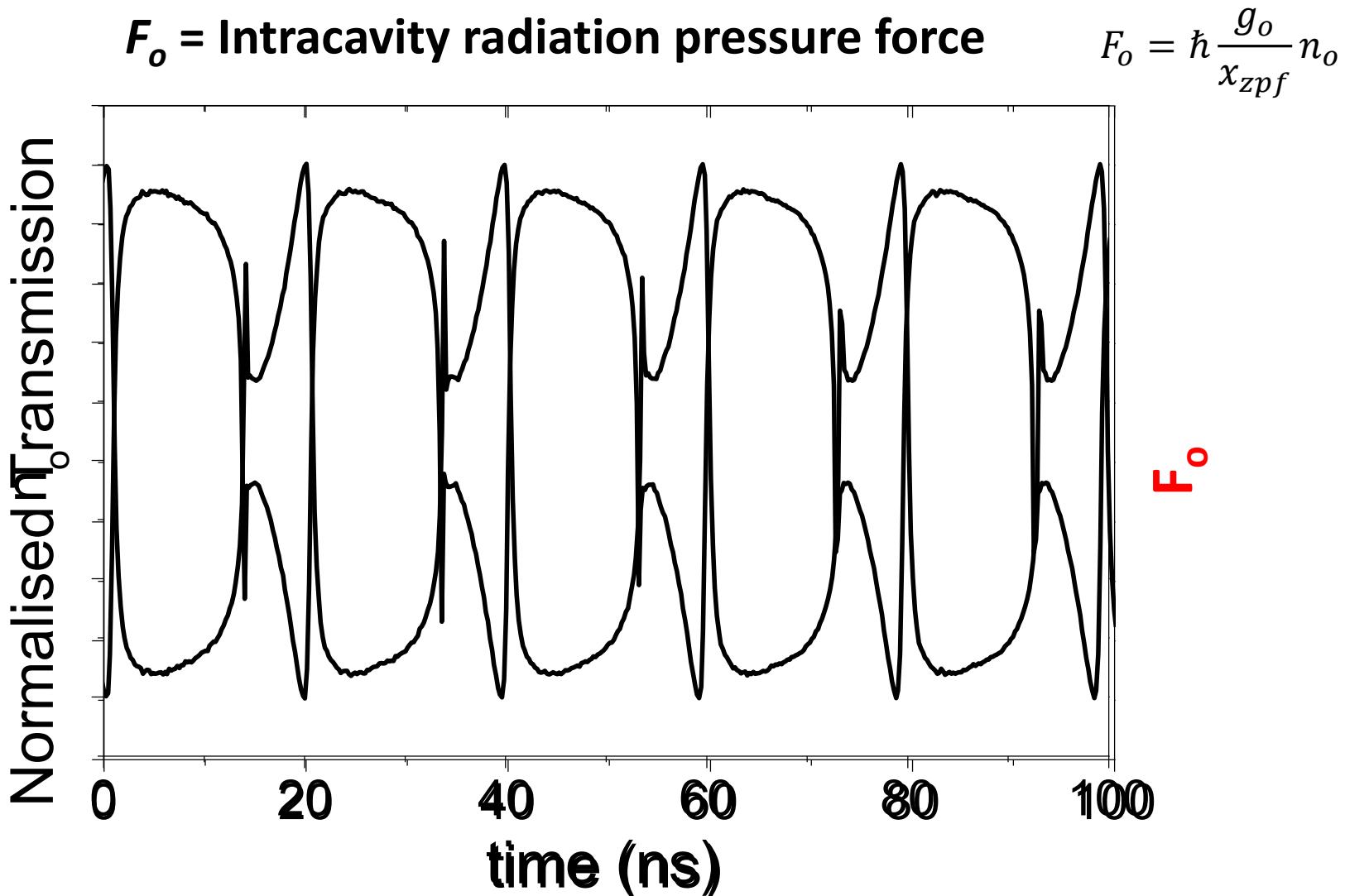
Knob



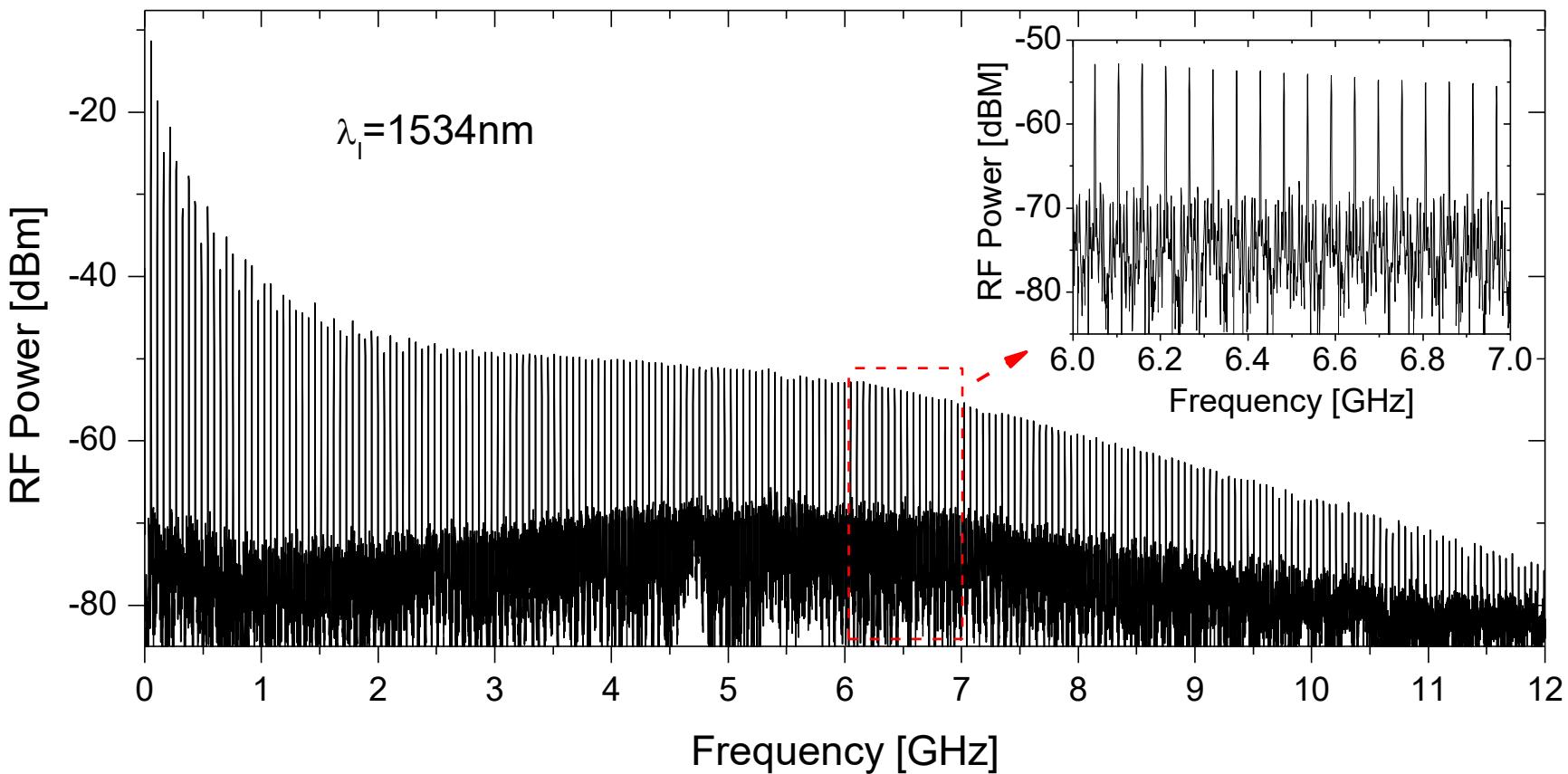
T. J. Johnson et al., Opt. Express, 14, 817-831 (2006)

A stable anharmonic trajectory of the optical resonance is established around the laser wavelength.

# Influence of the self-pulsing on the radiation pressure force



# RF signal above threshold



# “Phonon lasing” by exploiting the self-pulsing

Periodic/anharmonic intracavity radiation pressure force:

$$F_o = F_o(v_{SP}, 2v_{SP}, 3v_{SP}, \dots, Mv_{SP})$$

$$\nu_{SP} \neq \frac{\Omega_m}{M}$$

Since the structure is an OM crystal



Harmonic oscillator driven by the optical force

$$m_{eff} \ddot{x}_{zpf} + \omega_{m,i}^2 x_{zpf} = -\frac{\hbar}{m_o} \ddot{x}_{zpf}$$

$$\frac{dN}{dt} = -\frac{1}{\tau_{FC}} N + \beta \left( \frac{hc^3}{n^2 \lambda_o V_o} \right) n_o^2$$

$$\frac{d\Delta T}{dt} = -\frac{1}{\tau_T} \Delta T + \alpha_{FC} N n_o$$

$$n_o = n_{o,max} \left( \frac{\Delta \lambda_o^2}{4(\lambda_l - \lambda_r)^2 + \Delta \lambda_o^2} \right)$$

$$\lambda_r \approx \lambda_o - \frac{\partial \lambda_r}{\partial N} N + \frac{\partial \lambda_r}{\partial T} \Delta T + \frac{\lambda_o^2 g_o}{2\pi c x_{zpf}} u$$

OM

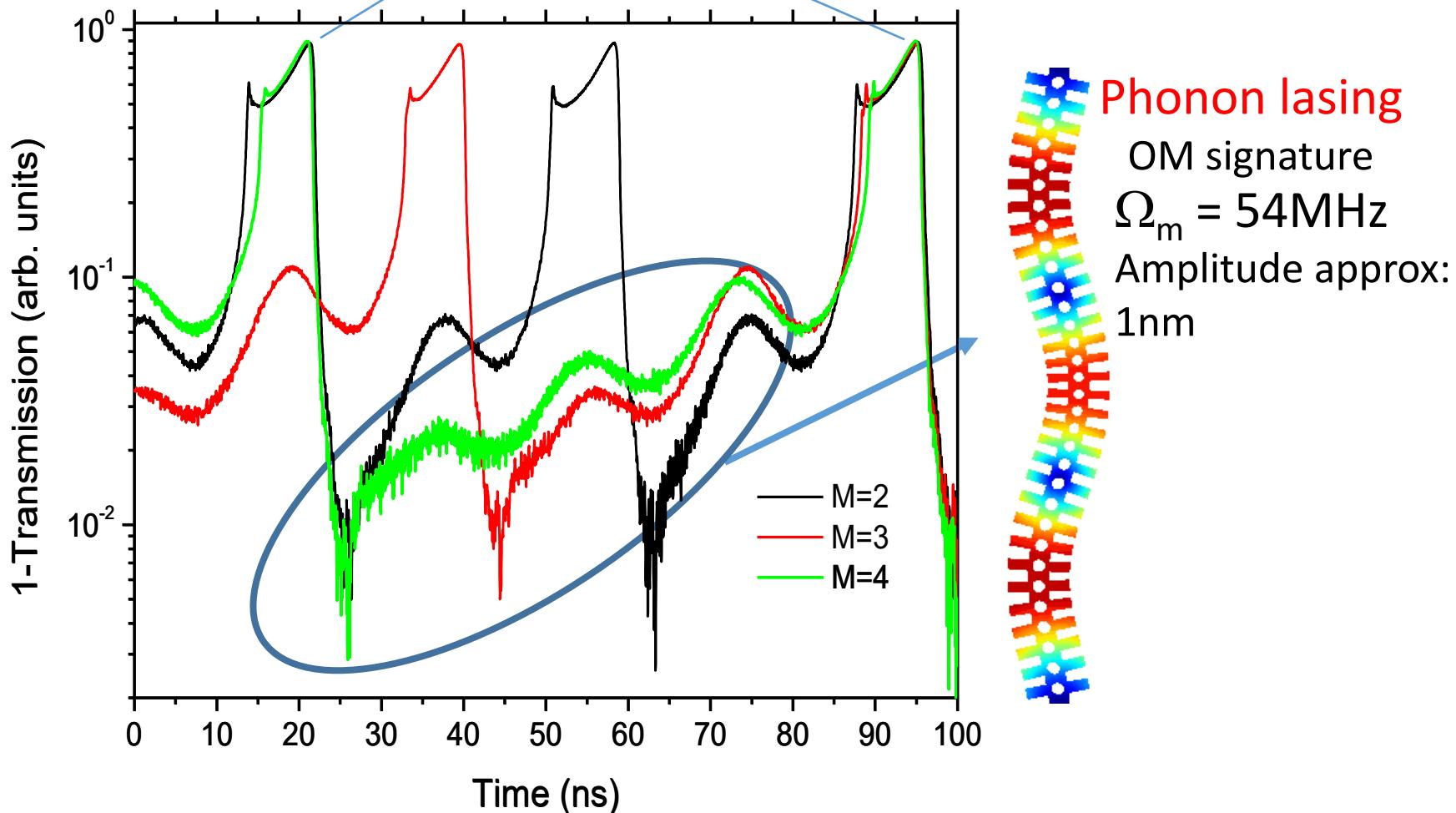
# Transmitted signal in the oscilloscope

If:

$$\nu_{SP} = \frac{\Omega_m}{M}$$

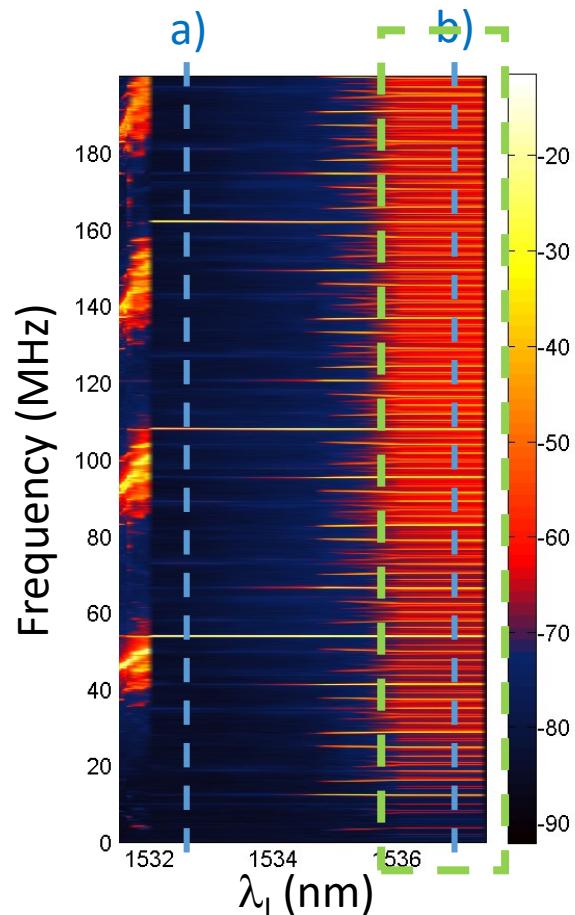
SP+OM signature

D. Navarro-Urrios et al.,  
Scientific Reports 5, 15733 (2015).



# Chaotic regime in the self-pulsing when coupled to the mechanics

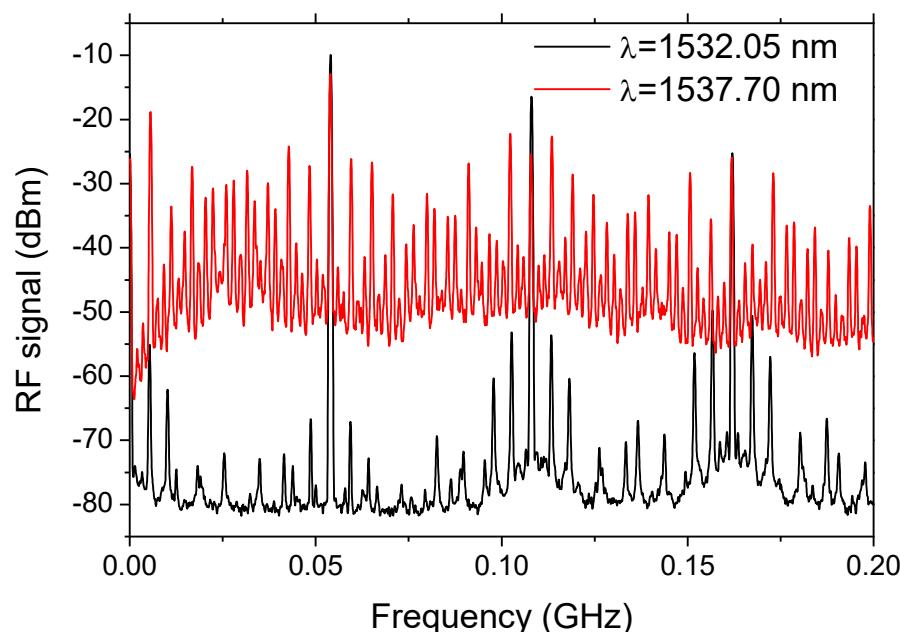
**Poincarè-Bendixson theorem:** No chaos in 2-dimensional, 1<sup>st</sup> order non-linear systems.  
-> Isolated self-pulsing cannot support chaotic trajectories.



$$\frac{dN}{dt} = -\frac{1}{\tau_{FC}} N + \beta \left( \frac{hc^3}{n^2 \lambda_o V_o^2} \right) n_o^2;$$

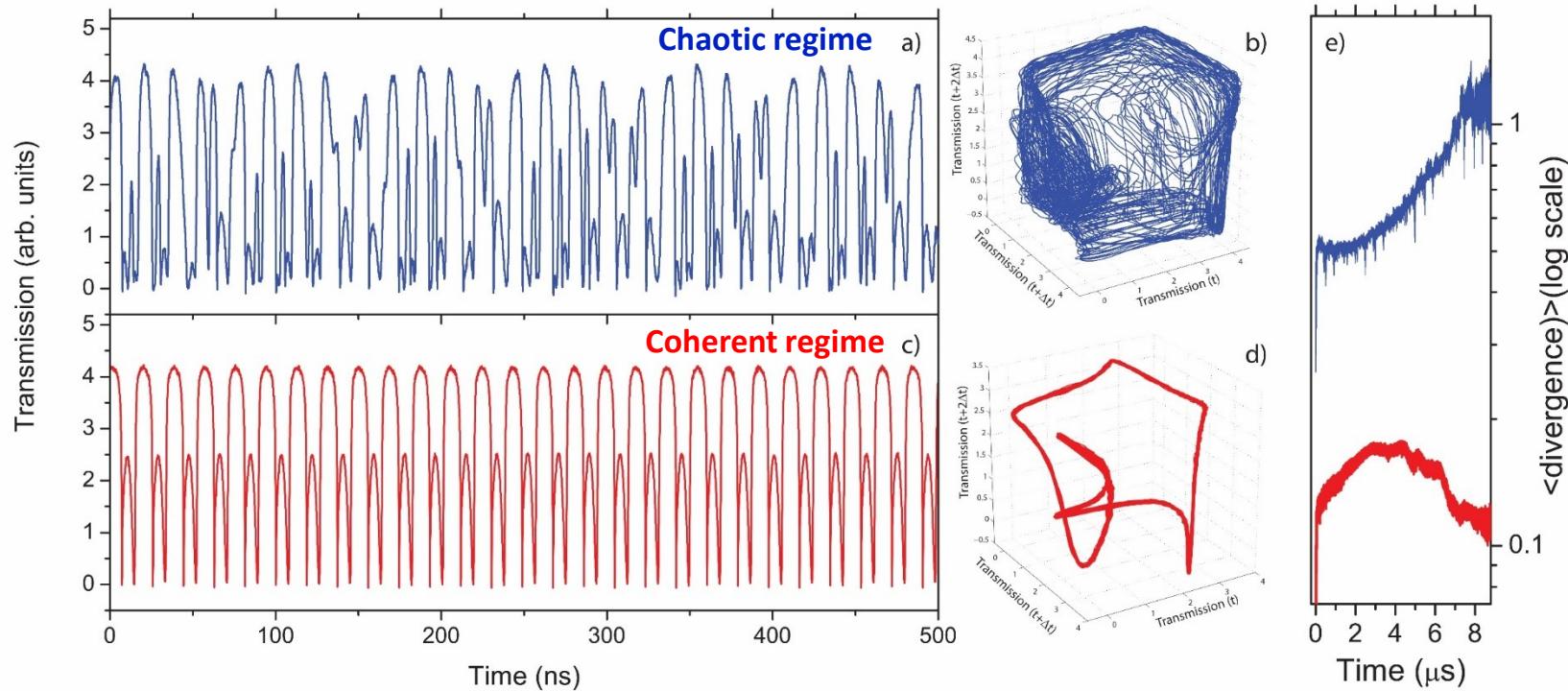
$$\frac{d\Delta T}{dt} = -\frac{1}{\tau_T} \Delta T + \alpha_{FC} N n_o;$$

$$m_{eff} \dots \\ \mathcal{L}_{m,i} \dots \\ \vec{\gamma}_o = \frac{\hbar}{x_{zpf}}$$



# Chaotic regime in the time domain

Time series (experimental) before and after the bifurcation leading to chaos

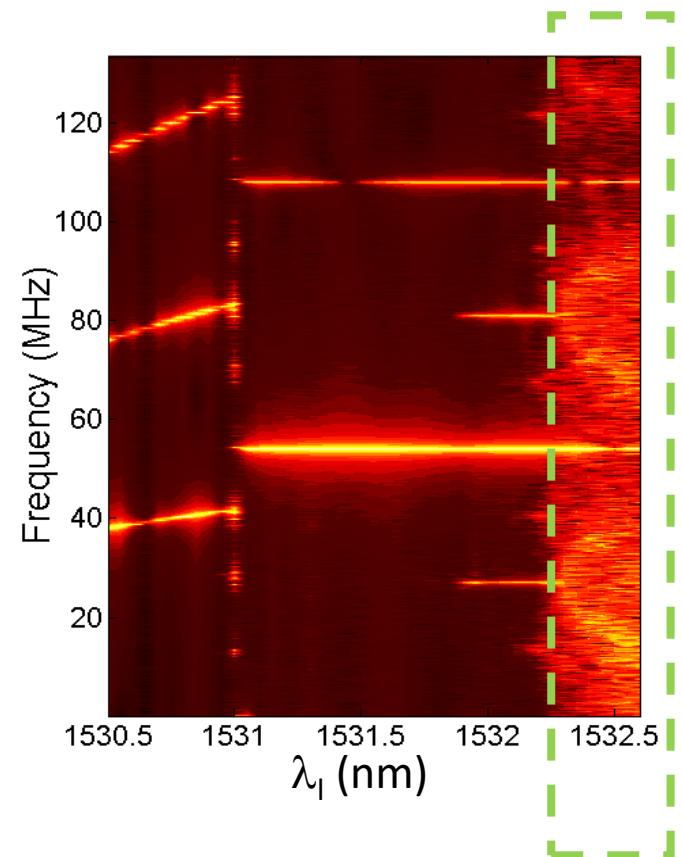
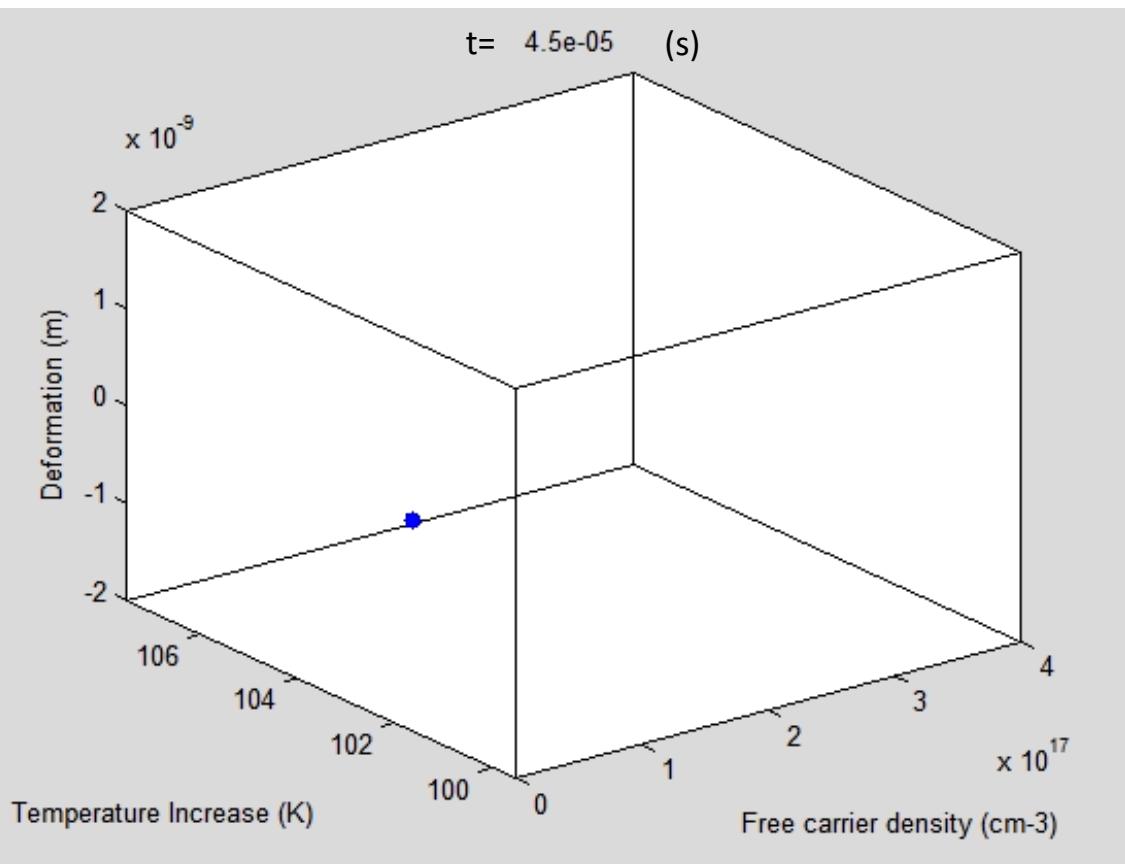


Largest Lyapunov Exponents greater than 1

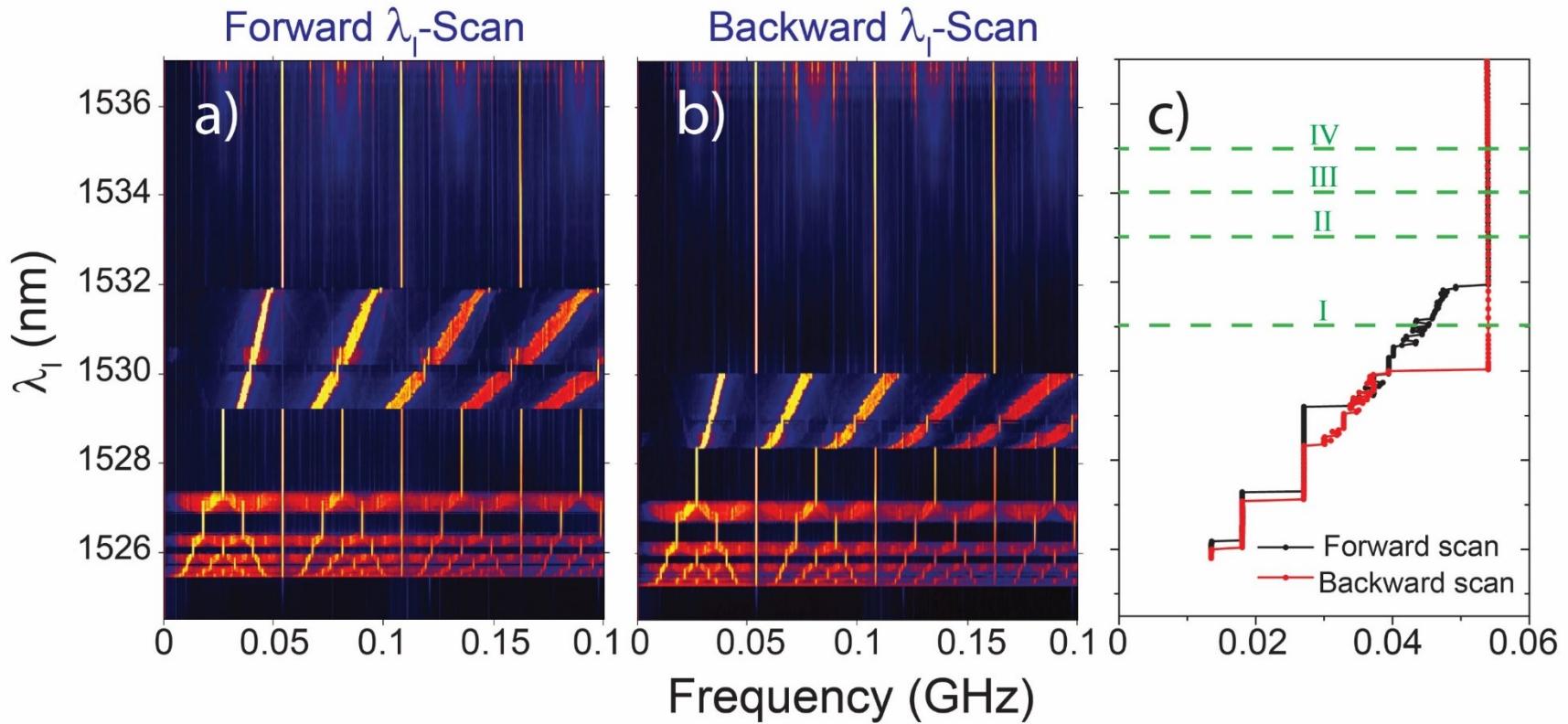
D. Navarro-Urrios *et al.*, Nature Communications 8, 14965, (2017).

# Chaotic regime

Numerical simulations ( $g_0/2\pi=100\text{KHz}$ )

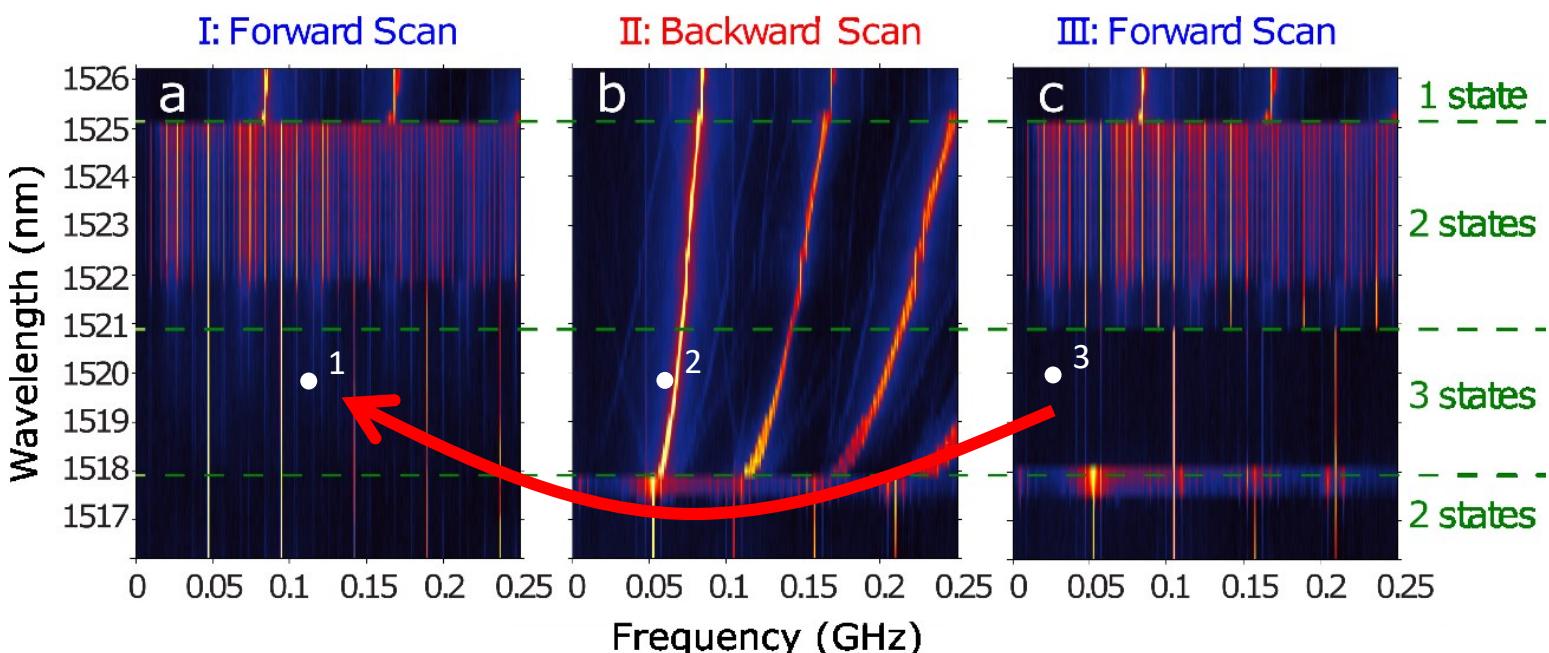
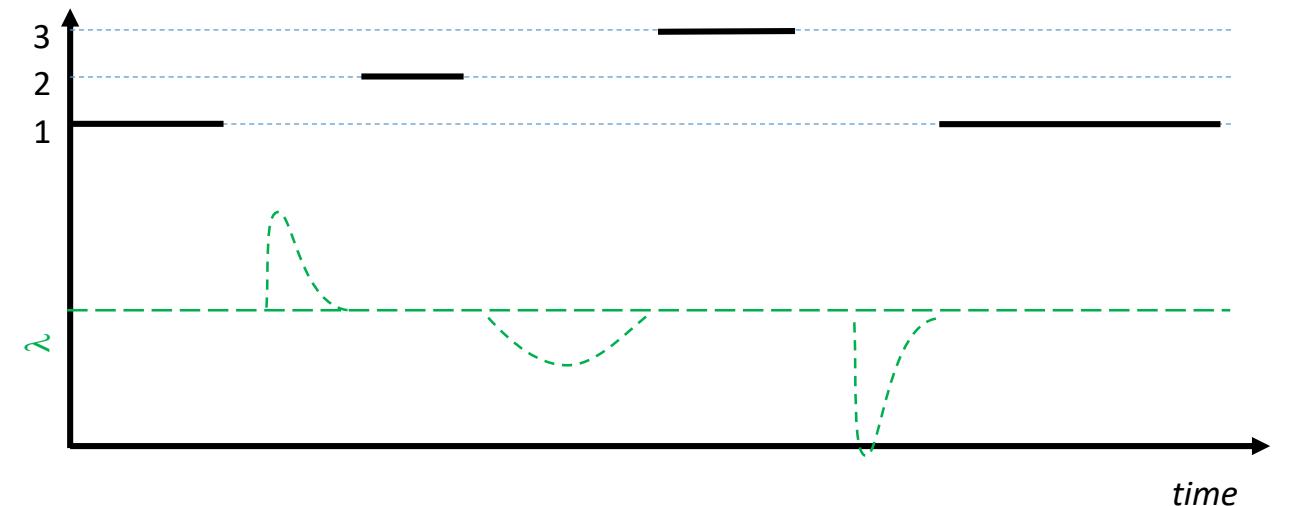


# Bistability (Experiment)

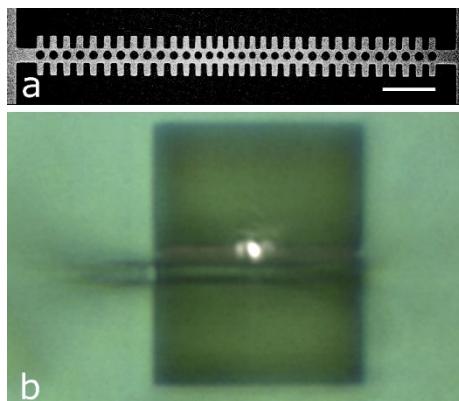
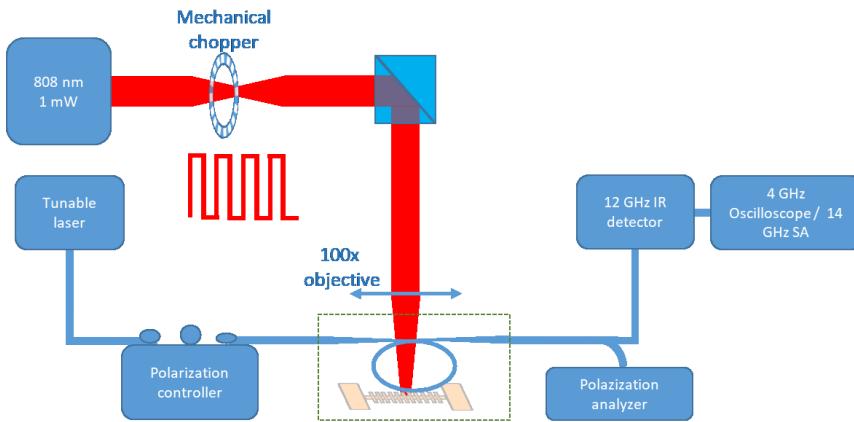


Backwards path: mechanical coherent oscillation is active and the feedback is strong enough to force self-pulsing frequency

# Multistability



# Switching between different dynamical states using an external source.

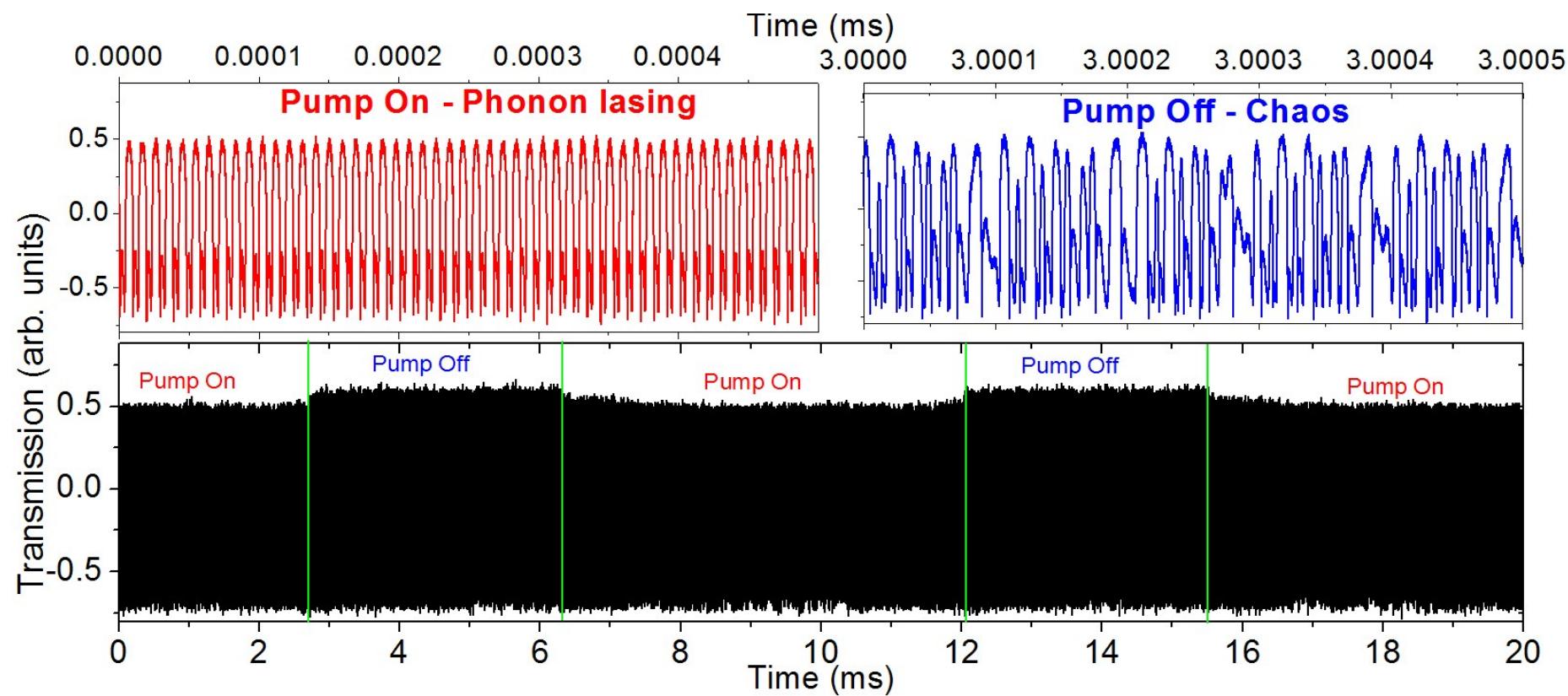


Optical image showing the OM crystal with the fiber below and the pump laser spot. The length of the nanobeam is 16.5  $\mu\text{m}$ .

*J. Maire, D. Navarro-Urrios, under review.*

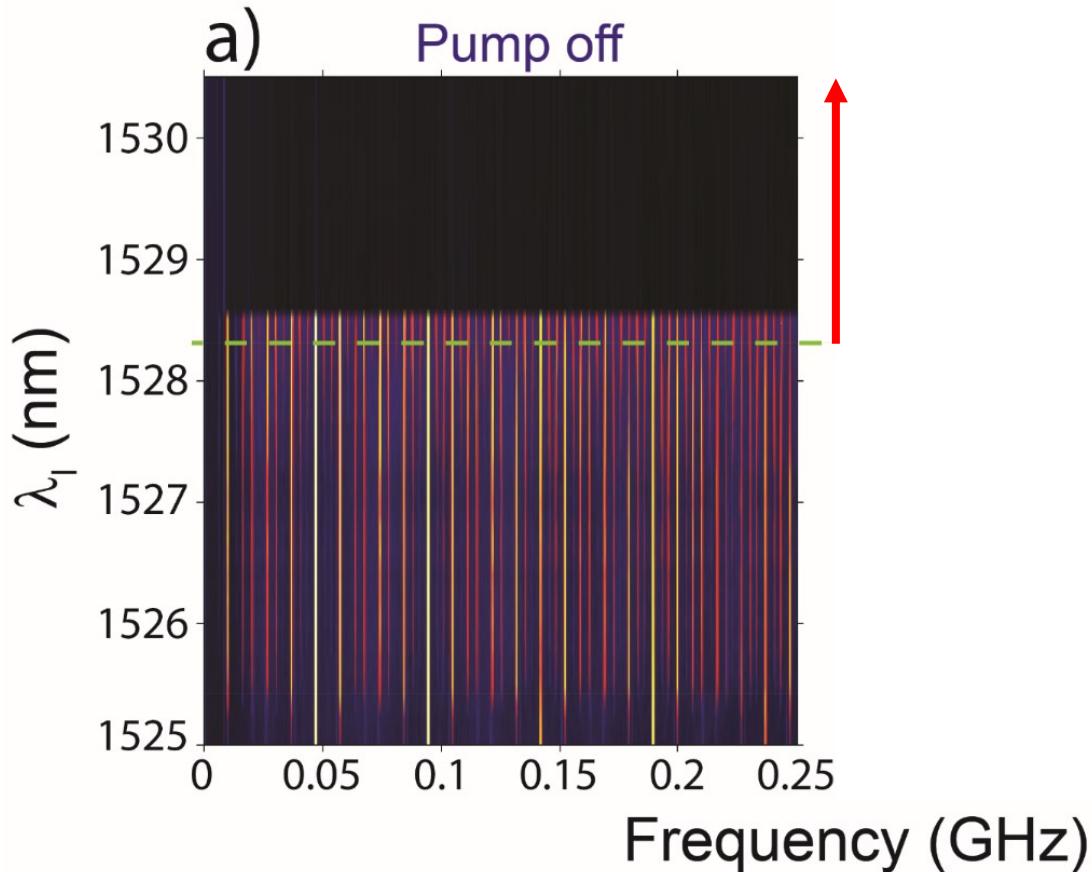


# Switching between different dynamical states using an external source.



# Switching mechanism

RF spectra with pump off and on (green line indicates the conditions of the measurements shown in the next slide).

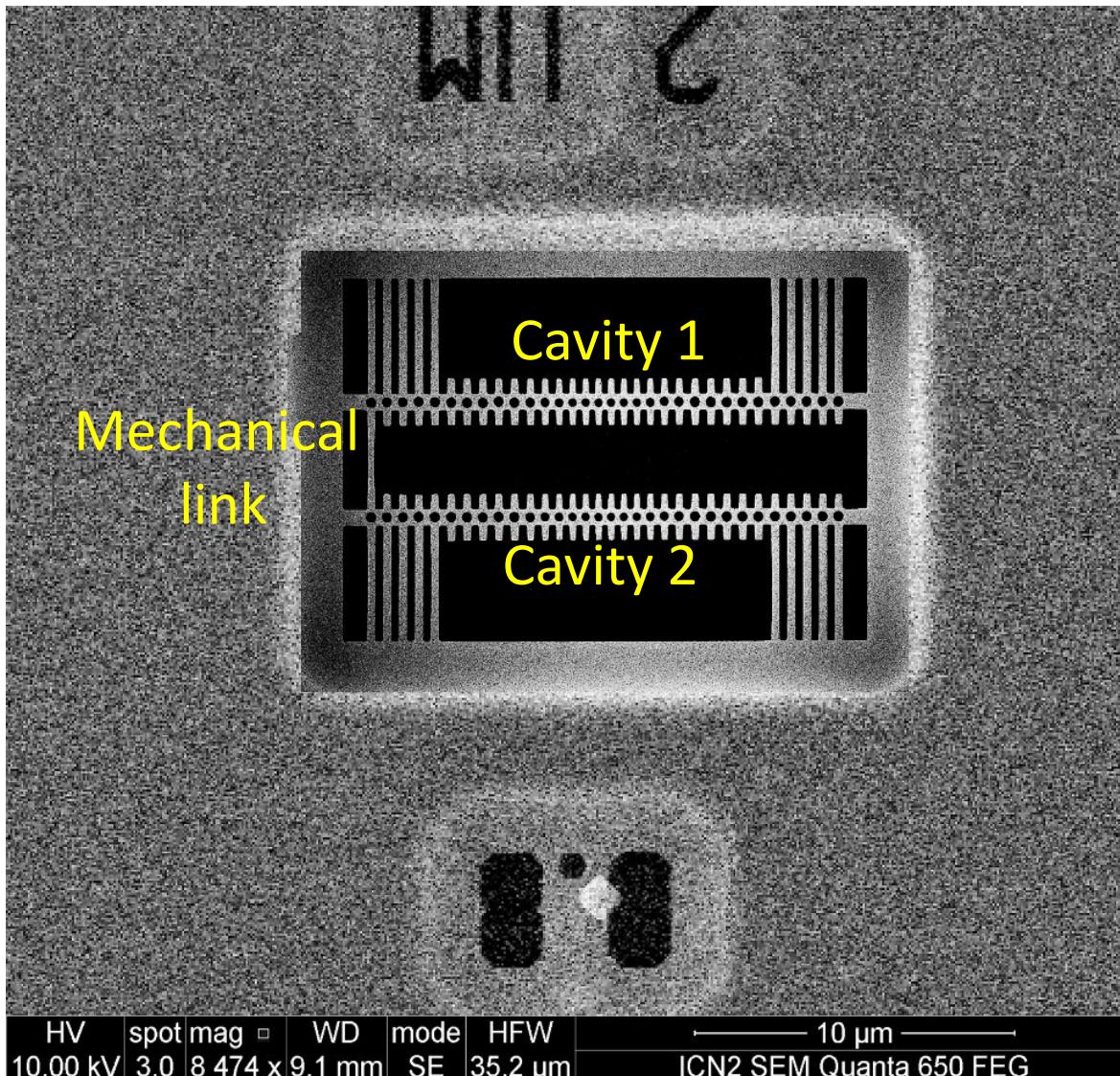


The top laser heats the OM cristal, inducing a spectral shift towards longer wavelengths

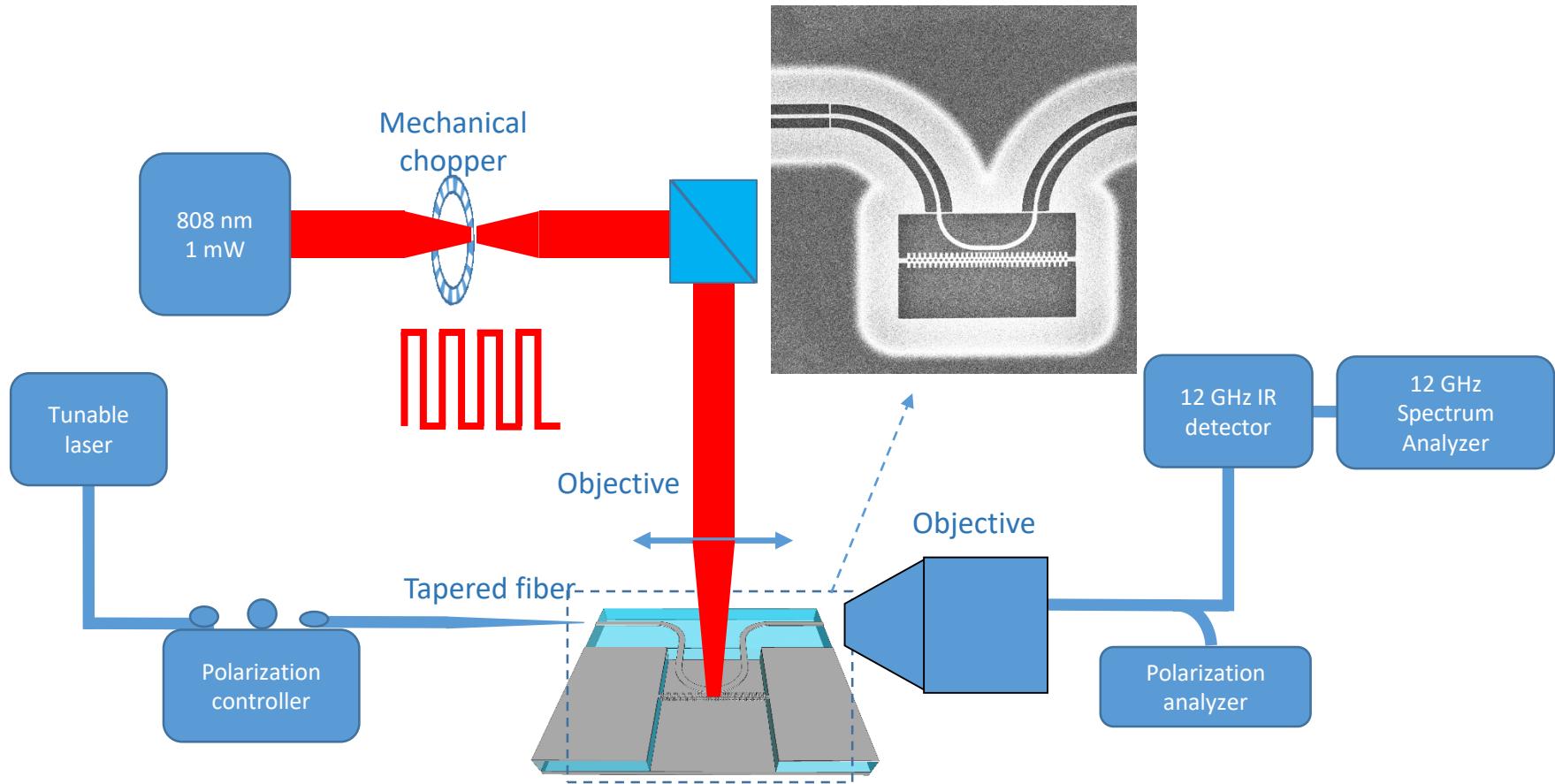
# Conclusions

- Rich set of **complex dynamical solutions, including chaos**, experimentally observed in a single and compact Si-based OM nanobeam, optically pumped with a continuous-wave, low-power laser source.
- **Hysteresis, bistability and tristability** of different kinds involving self-pulsing, phonon “lasing” and chaos.
- **Switching between different dynamic solutions** using an external heating source

# Outlook: Coupled cavities



# Outlook: Towards integration



# Thank you!

Ramón y Cajal: RYC-2014-15392



FET-Open H2020-EU-713450

