

Quantum transport in twisted bilayer graphene

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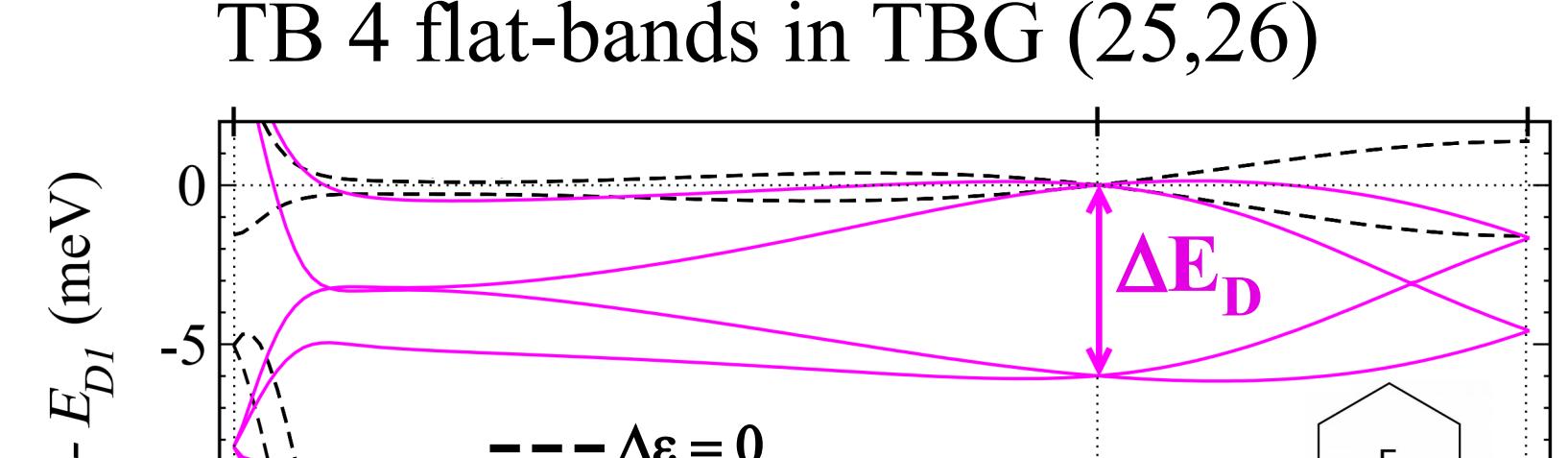
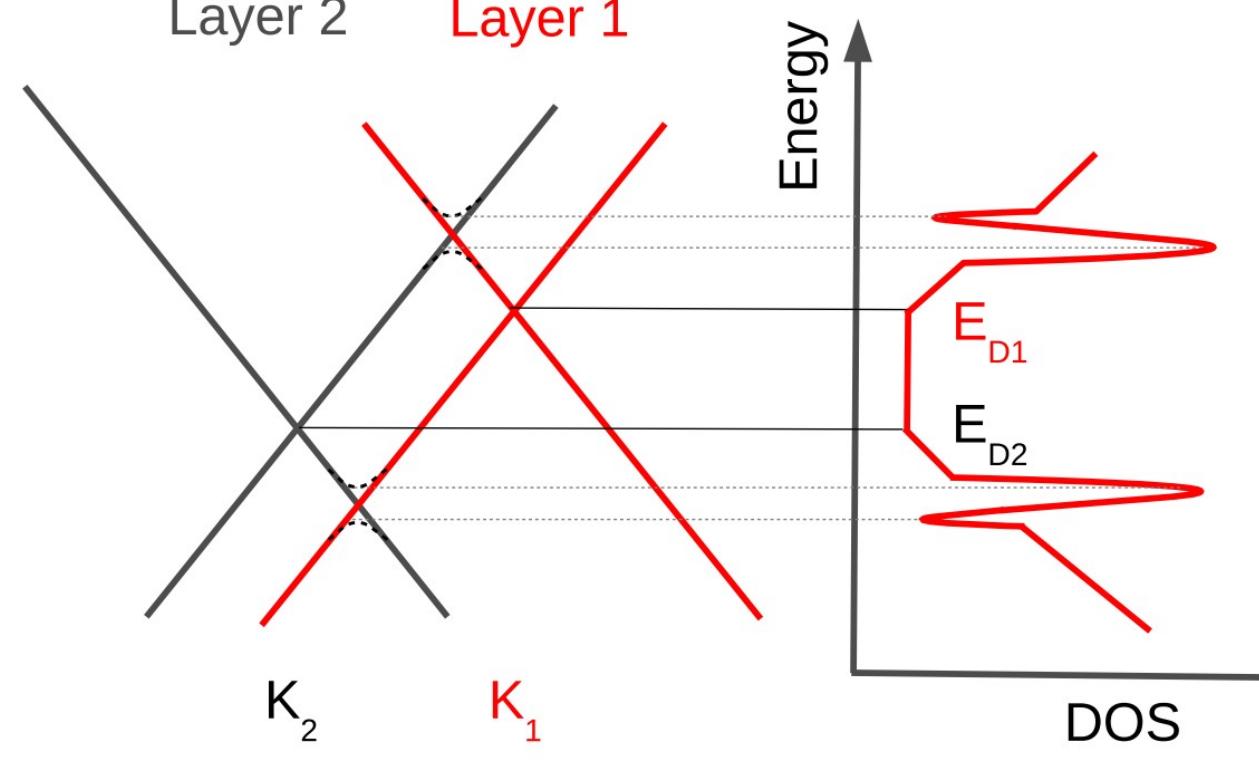
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ABSTRACT: It has been shown theoretically and experimentally that twisted bilayer graphene (TBG), forming Moiré patterns, confine electrons in a tunable way as a function of the rotation angle [1-3]. The discovery of correlated insulators and superconductivity in 2018 [3] at so-called “magic angles” has stimulated an avalanche of experimental and theoretical activities. In the framework of the Kubo-Greenwood formula for the conductivity, we present tight-binding (TB) calculations of quantum diffusion properties in TBG at various rotation angles θ [4,5]. We analyze in particular the effect of static defects, the effect of an electric bias. One of the main results is that flat-bands induce a breakdown of the standard Boltzmann theory of transport.

This anomalous quantum transport in flat-bands can exist in other systems like Quasicrystals [6] and flat-bands induced by defects in graphene [7,8].

Effect of an electric bias on bands [4]

A difference $\Delta\epsilon$ between the on-site energies of layer 1 and layer 2
→ Difference ΔE_D between the two corresponding Dirac energies



Twisted bilayer graphene (TBG)
with varius rotation angle θ

| (n,m) | θ ($^{\circ}$) | N |
|---------|-------------------------|-------|
| (1,3) | 32.20 | 52 |
| (5,9) | 18.73 | 604 |
| (2,3) | 13.17 | 76 |
| (3,4) | 9.43 | 148 |
| (6,7) | 5.08 | 508 |
| (8,9) | 3.89 | 868 |
| (12,13) | 2.65 | 1876 |
| (15,16) | 2.13 | 2884 |
| (25,26) | 1.30 | 7804 |
| (33,34) | 0.99 | 13468 |

N: number of atoms in a Moiré cell

⇒ Reduction of band velocity by bias potential

Microscopic conductivity (static defects) [5]

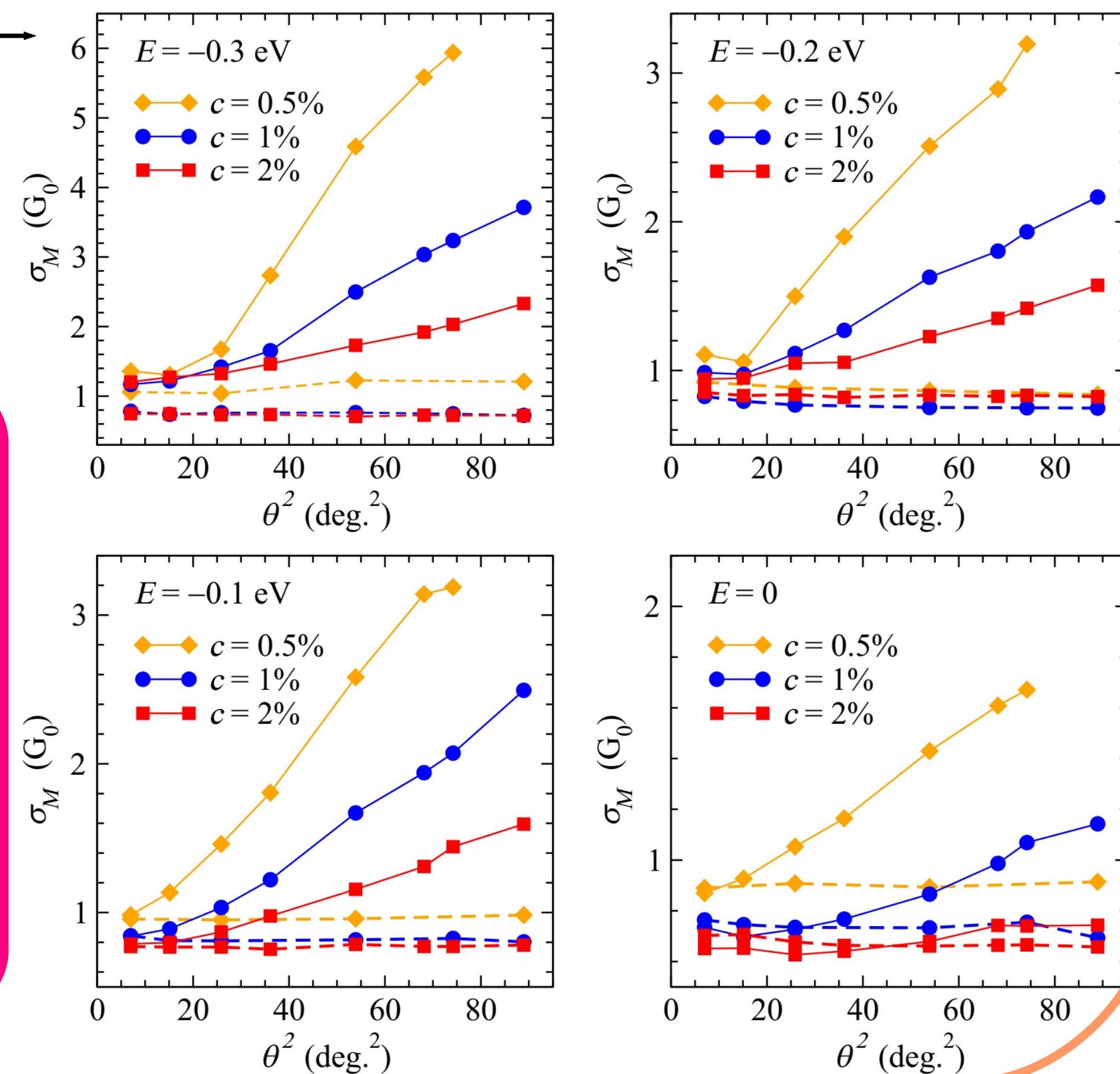
Microscopic conductivity σ_M is a good estimation of room temperature conductivity (without quantum corrections)

With defects (vacancies) located in layer 2 only:

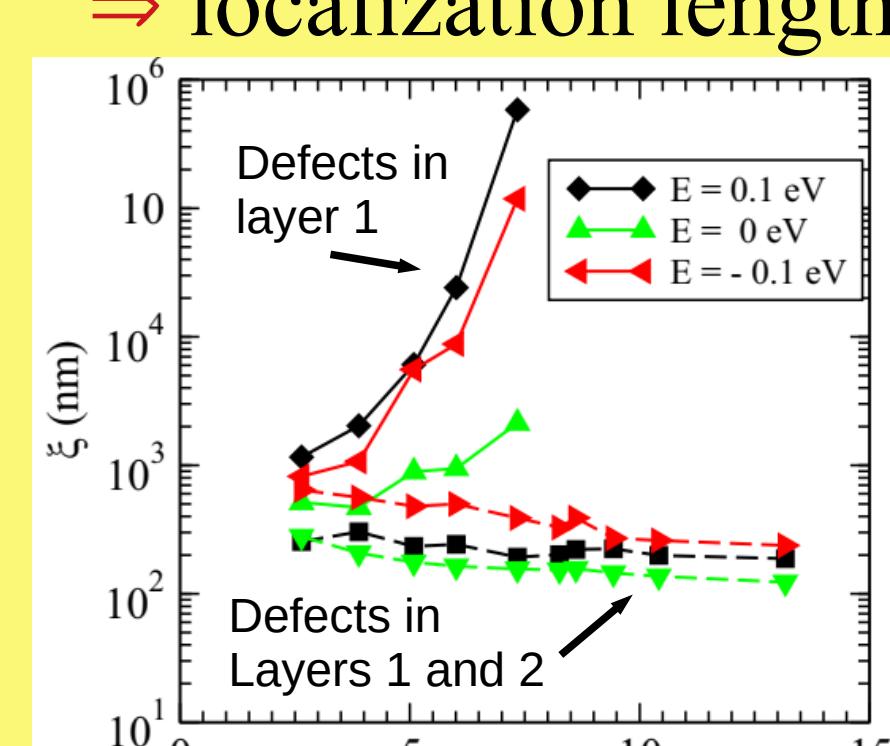
$$\text{Continuous model: } \sigma_M(E) \simeq \sigma_{M,\text{MLG}} \left(1 + \frac{\rho_1(E) \theta^2}{\rho_2(E) \theta_0^2} \right) \quad \rho_i \text{ DOS layer } i \quad \theta_0 \approx 2^\circ$$

TB calculations:

For Fermi energy E closed to Dirac energy ($E_D = 0$), σ_M is almost linear with θ^2 .



At low temperature:
quantum corrections
→ localization length



Quantum diffusion: Boltzmann and non-Boltzmann terms [4]

Kubo-Greenwood dc-conductivity $\sigma_{xx}(E_F, \tau) = \frac{e^2}{S} n(E_F) \mathcal{D}(E_F, \tau)$

Diffusivity $D(E_F, \tau) = \frac{1}{2\tau^2} \int_0^\infty \Delta X^2(E_F, t) e^{-t/\tau} dt$

Relaxation time approximation: Scattering time (defect or phonon): τ

Mean square spreading: $\Delta X^2(E, t) = \langle (X(t) - X(0))^2 \rangle_E$

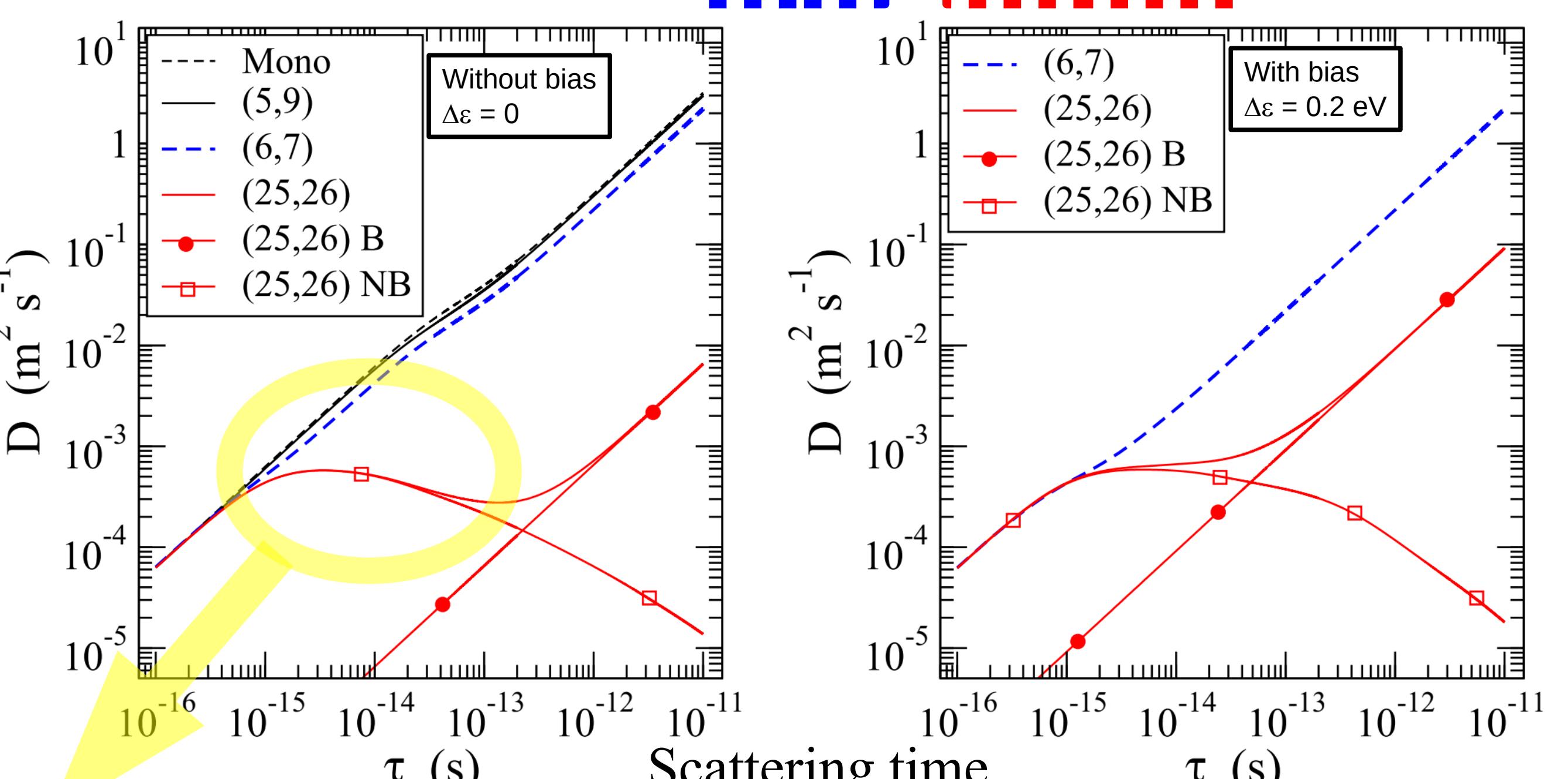
$$\Delta X^2(E, t) = 2\hbar^2 \left\langle \sum_{n'} \left[1 - \cos((E_n - E_{n'}) \frac{t}{\hbar}) \right] \frac{|\langle n\vec{k}|V_x|n'\vec{k}\rangle|^2}{(E_n - E_{n'})^2} \right\rangle_{E_n=E} = V_B^2 t^2 + \Delta X_{NB}(E, t)$$

Velocity: $V_x = \frac{1}{i\hbar} [X, H]$ → Diagonal terms (Boltzmann): intra-bands
→ Non diagonal terms (Non Boltzmann): inter-bands

⇒ For Twisted Bilayer Graphene (25,26), $\theta = 1.3^\circ$, close to magic angle, Non-Boltzmann terms dominate in the conductivity

Diffusivity of the 4 flat-bands of the Moiré (half-filling)

$$\mathcal{D}(E_F, \tau) = \mathcal{D}_B(\tau) + \mathcal{D}_{NB}(E_F, \tau)$$



SUMMARY:

- A applied bias potential –or and an asymmetric doping between the two layers– reduces the velocity of Dirac-bands in TBG
- Conductivity of TBG, with an asymmetric distribution of states defects between the two layers, varies like θ^2
- Non-Boltzmann terms (inter-band hopping terms) dominate in the quantum diffusion in flat-bands in TBG

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