

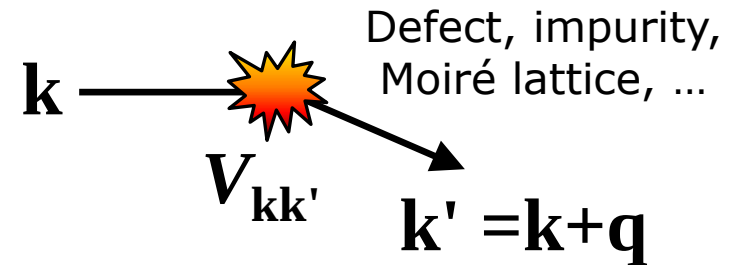
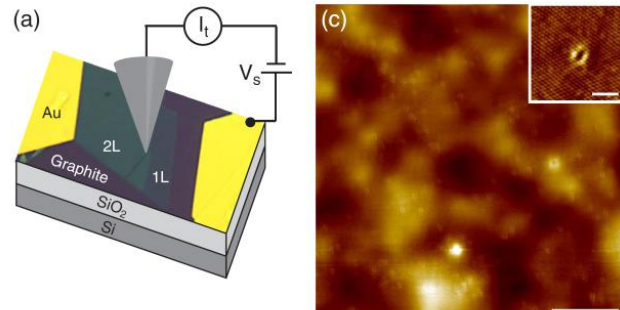
# **A unified approach to quasiparticle interference in two-dimensional materials**

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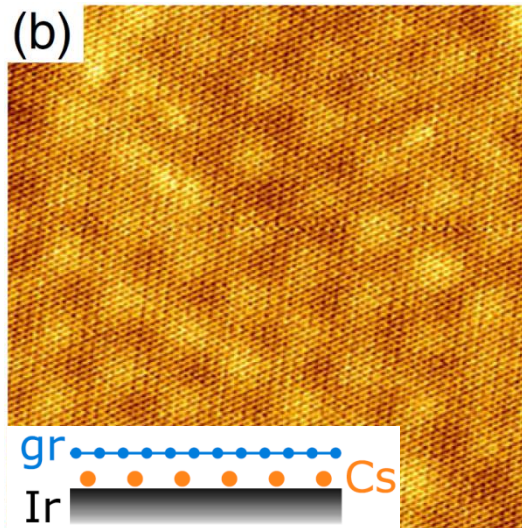
Dept. of Micro and Nanotechnology  
Technical University of Denmark

Graphene 2018, Dresden 06/2018

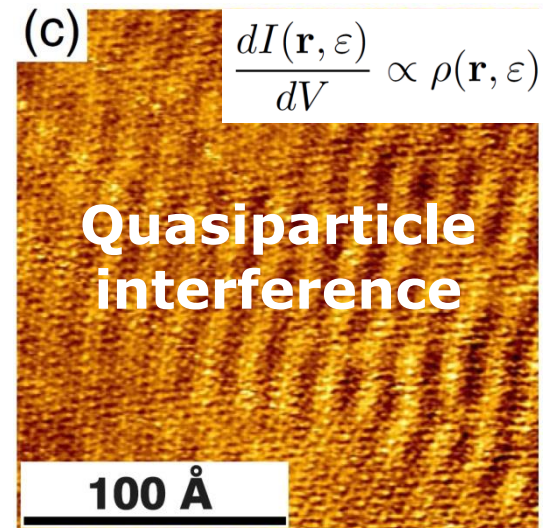
# Scanning tunneling spectroscopy



Topography  
(const. current)



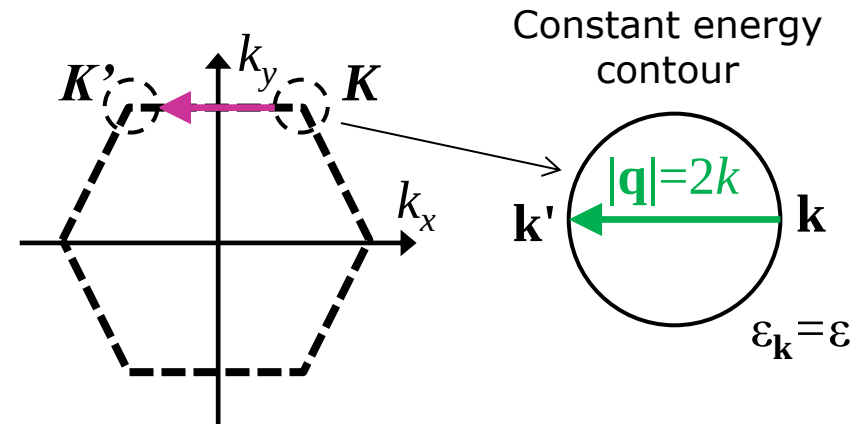
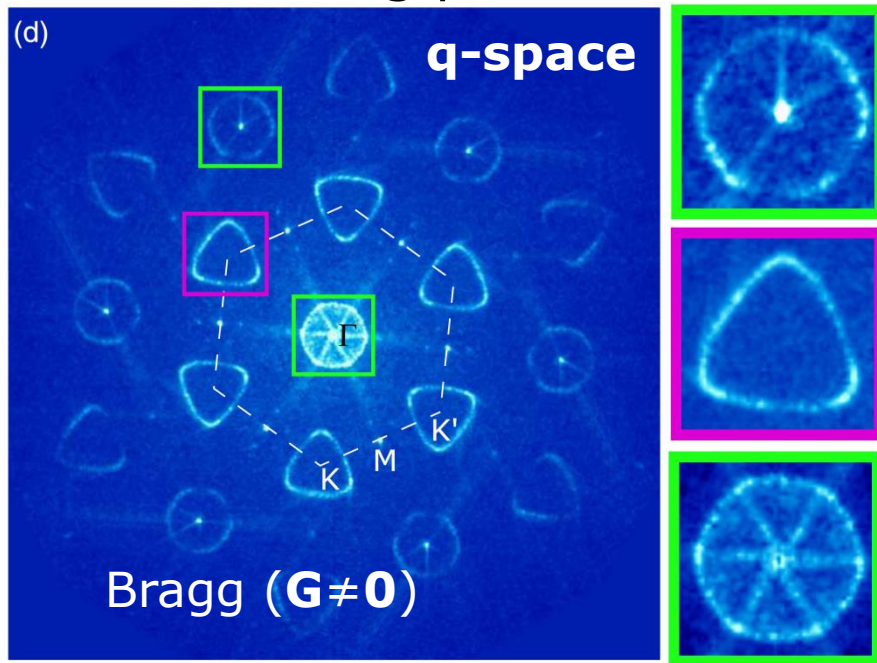
Spectroscopy



# Fourier transform STS (FT-STS)

$$\rho(\mathbf{q} + \mathbf{G}, \varepsilon) = \int d\mathbf{r} e^{-i(\mathbf{q} + \mathbf{G}) \cdot \mathbf{r}} \rho(\mathbf{r}, \varepsilon)$$

## Scattering processes



- Scattering (elastic) on constant energy contours in  $\mathbf{k}$ -space.
- Nesting vectors (backscattering) dominate  $\rightarrow$  Friedel osc.
- Replicas outside 1<sup>st</sup> BZ (Bragg).

• JDOS:  $\rho_{\text{JDOS}}(\mathbf{q}, \varepsilon) \propto \sum_{\mathbf{k}} \rho_{\mathbf{k}}(\varepsilon) \rho_{\mathbf{k}+\mathbf{q}}(\varepsilon)$

FIG. 1. Scattering patterns in strongly doped graphene. (a) Inverted LEED pattern of gr/Cs/Ir(111) at  $E = 143.5$  eV, the intercalated Cs forms a  $(2 \times 2)_{\text{or}}$  superstructure. The sketch

PRL **118**, 116401 (2017)

# A “unified” FT-STs theory

LDOS:  $\rho(\mathbf{r}, \varepsilon) = -1/\pi \text{Im} G(\mathbf{r}, \mathbf{r}; \varepsilon)$

FT of LDOS:

$$\rho(\mathbf{q} + \mathbf{G}, \varepsilon) = -\frac{1}{2\pi i} \sum_{mn} \sum_{\mathbf{k}} n_{\mathbf{k}, \mathbf{q}}^{mn}(\mathbf{G}) \times [G_{\mathbf{k}+\mathbf{q}, \mathbf{k}}^{nm}(\varepsilon) - G_{\mathbf{k}, \mathbf{k}+\mathbf{q}}^{mn}(\varepsilon)^*]$$

Interference in  $q$  space

$$n_{\mathbf{k}, \mathbf{q}}^{mn}(\mathbf{G}) = \langle \psi_{m\mathbf{k}} | e^{-i(\mathbf{q}+\mathbf{G}) \cdot \hat{\mathbf{r}}} | \psi_{n\mathbf{k}+\mathbf{q}} \rangle$$

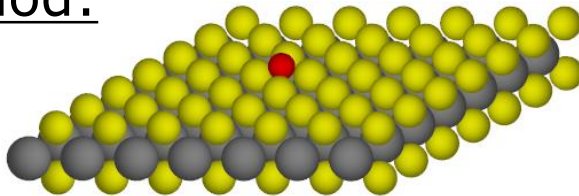
(pristine)

Scattering processes

$$V_{\mathbf{k}\mathbf{k}'}^{mn} = \langle \psi_{m\mathbf{k}} | \hat{V} | \psi_{n\mathbf{k}+\mathbf{q}} \rangle$$

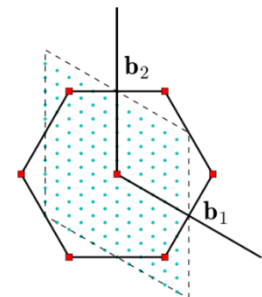
(defect/impurity)

DFT based method:



$$V_i(\mathbf{r}) = V_{\text{dis}}^i(\mathbf{r}) - V_{\text{pris}}(\mathbf{r})$$

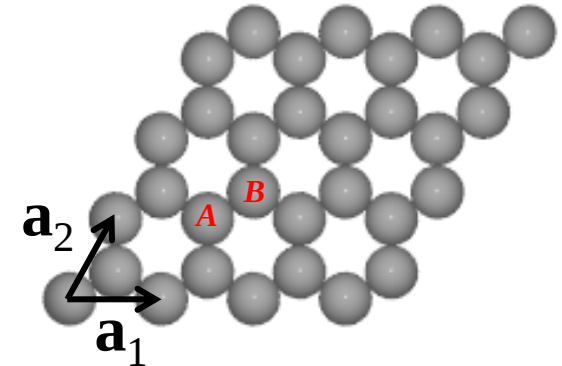
- Supercell
- LCAO basis
- Spin-orbit
- $T$ -matrix
- $N \times N$  BZ sampling



# Graphene tight-binding

$$H_0 = -t \sum_{\mathbf{k}} [\phi_{\mathbf{k}} c_{A\mathbf{k}}^\dagger c_{B\mathbf{k}} + \text{h.c.}]$$

$$\phi_{\mathbf{k}} = 1 + e^{i\mathbf{k} \cdot \mathbf{a}_1} + e^{i\mathbf{k} \cdot \mathbf{a}_2}$$

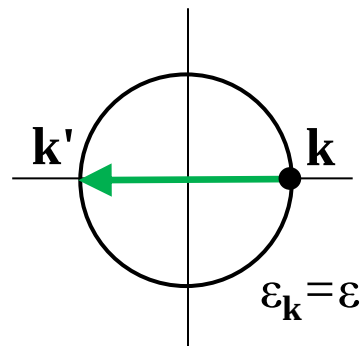


$$\mathbf{R}_n = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2$$

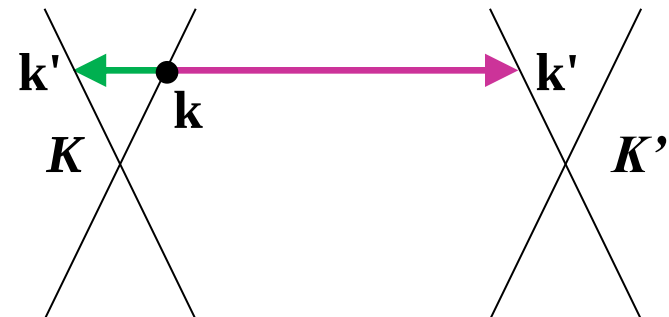
Impurity model:

$$\mathbf{V} = V_0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Breaks  $A, B$  sublattice symmetry  
 $\rightarrow q=2k$  backscattering allowed

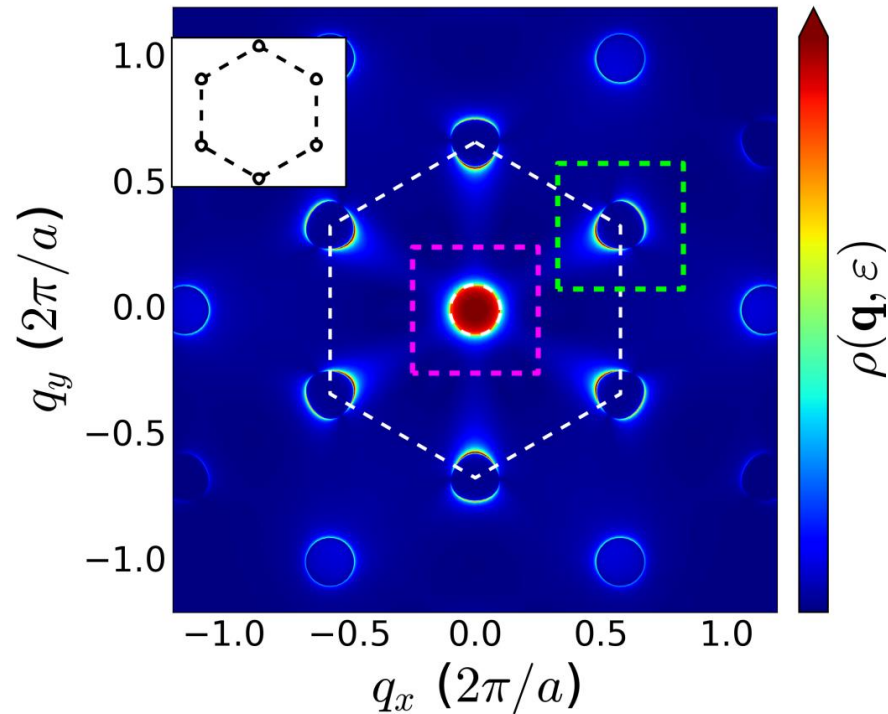


$\mathbf{k}, \mathbf{k}'$  independent  $\rightarrow$  intra + intervalley scattering

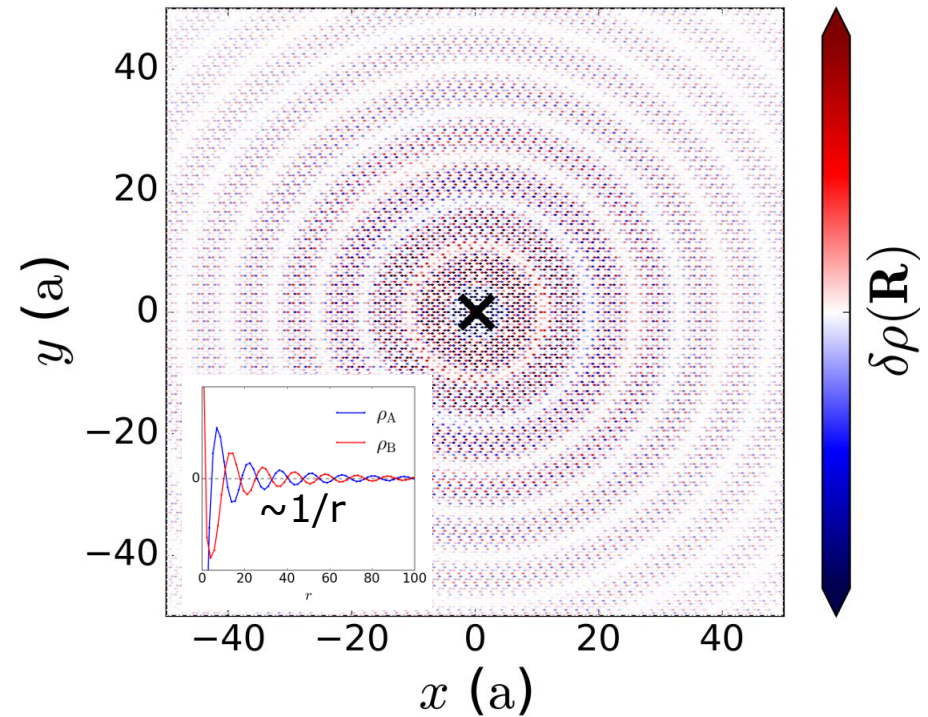


# Weak impurity

FT-STs

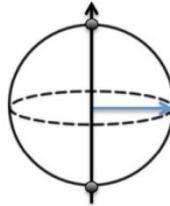


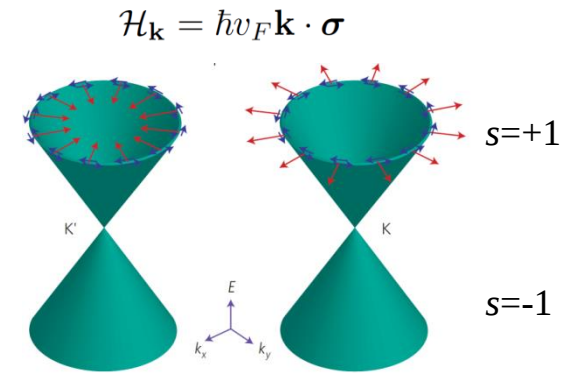
Friedel oscillations



- Filled disc instead of  $q=2k$  backscatter ring + strongly anisotropic intervalley features (PRL **100**, 076601 (2008)).
- Backscatter ring at Bragg points (weak intensity).

# Role of (intrinsic) chirality

$$\chi_{s\mathbf{k}} = (1, se^{i\theta_{\mathbf{k}}})^T =$$




Phase-factor element:

$$n_{\mathbf{k},\mathbf{q}}^{ss'}(\mathbf{G}) = \langle \psi_{s\mathbf{k}} | e^{-i(\mathbf{q}+\mathbf{G}) \cdot \hat{\mathbf{r}}} | \psi_{s'\mathbf{k}+\mathbf{q}} \rangle$$

$$\psi_{s\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{A}} e^{i\mathbf{k} \cdot \mathbf{r}} \chi_{s\mathbf{k}}$$

$$\rightarrow \chi_{s\mathbf{k}}^\dagger M(\mathbf{q} + \mathbf{G}) \chi_{s'\mathbf{k}+\mathbf{q}} \quad ; \quad M(\mathbf{q}) = \begin{pmatrix} e^{-i\mathbf{q} \cdot \mathbf{R}_A} & 0 \\ 0 & e^{-i\mathbf{q} \cdot \mathbf{R}_B} \end{pmatrix}$$

Intravalley ( $\mathbf{q} \sim 0$ )

$$M(\mathbf{q}) \approx \sigma_0$$

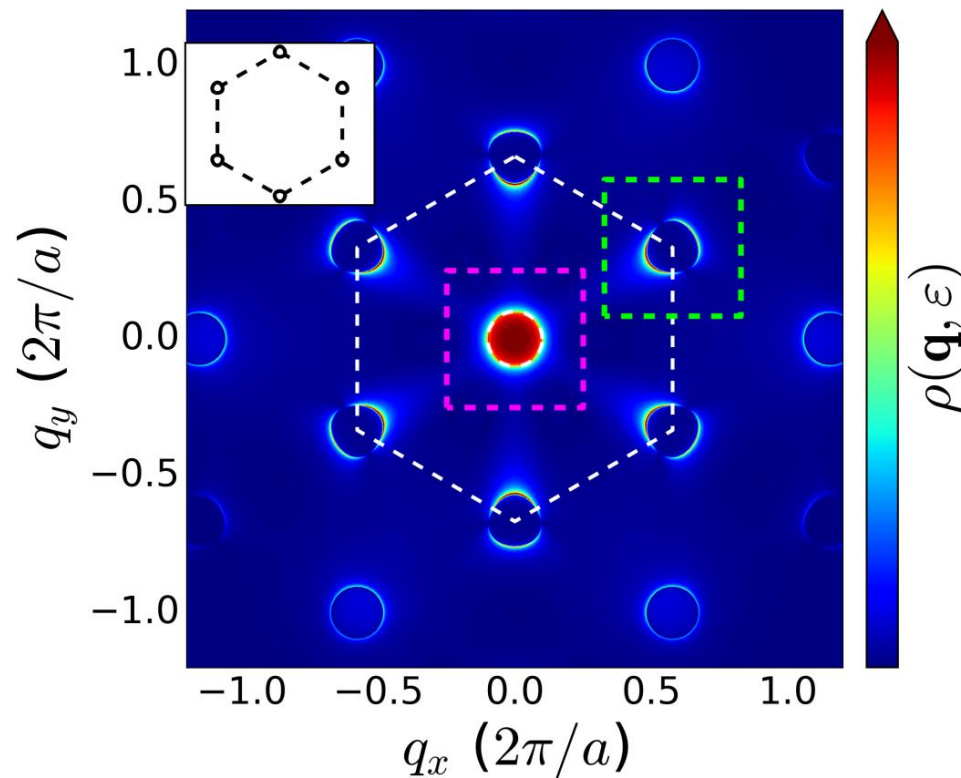
$$n_{\mathbf{k},\mathbf{q}}^{ss'} = \frac{1}{2} [1 + ss' \exp(i\theta_{\mathbf{k},\mathbf{k}+\mathbf{q}})]$$

Intervalley ( $\mathbf{q}=\mathbf{K},\mathbf{K}'$ )  
and Bragg ( $\mathbf{G} \neq 0$ )

$$M(\mathbf{q}) \neq \sigma_0$$

$$n_{\mathbf{k},\mathbf{q}}^{ss'} = F(\theta_{\mathbf{k}}, \theta_{\mathbf{k}+\mathbf{q}})$$

# Role of (intrinsic) chirality



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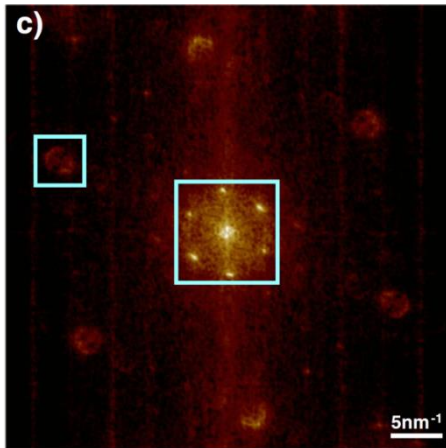
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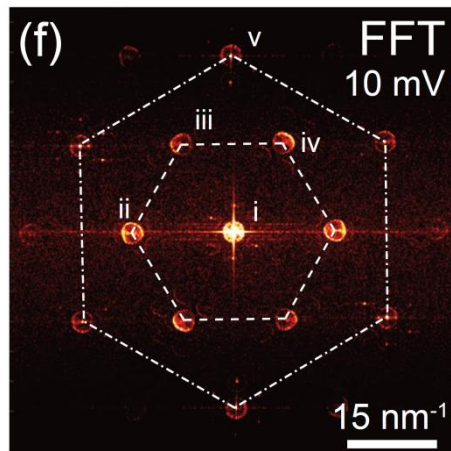
$$n_{\mathbf{k},\mathbf{q}}^{ss'} = F(\theta_{\mathbf{k}}, \theta_{\mathbf{k}+\mathbf{q}})$$

# QPI/FT-STs in graphene



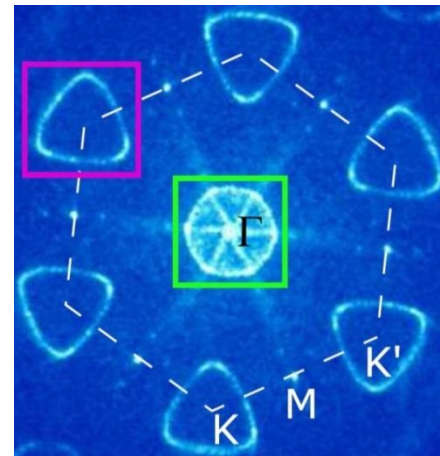
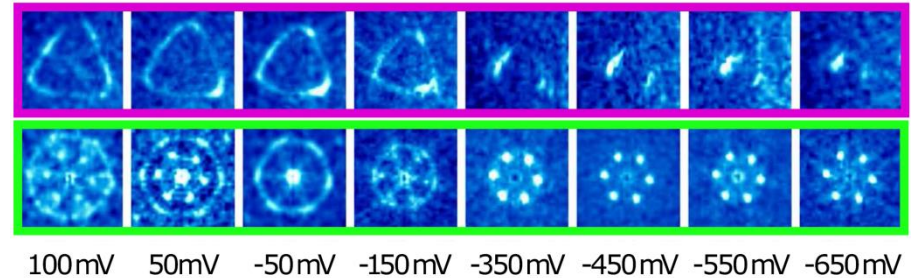
PRL **101**, 206802 (2008)

PHYSICAL REVIEW B **86**, 045444 (2012)

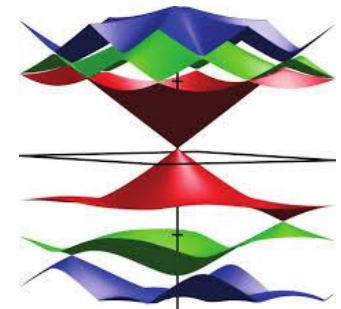


PHYSICAL REVIEW B **95**, 075429 (2017)

PRL **118**, 116401 (2017)



Moiré minibands



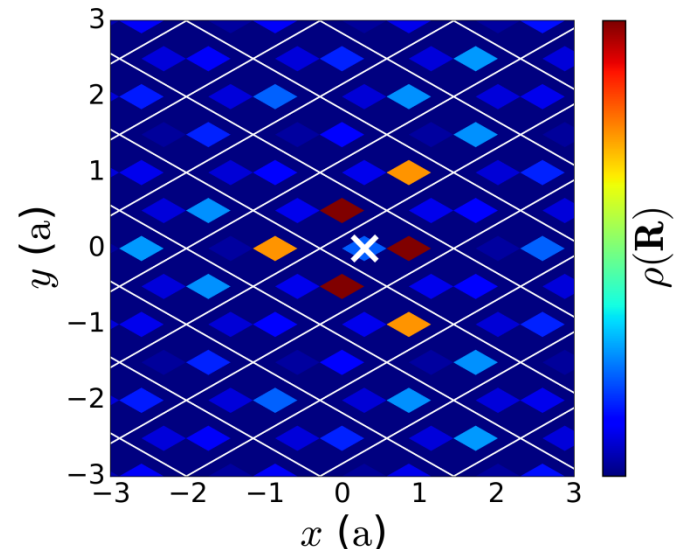
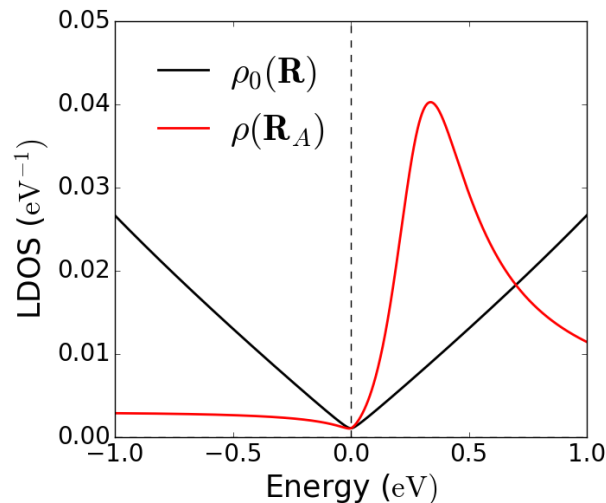
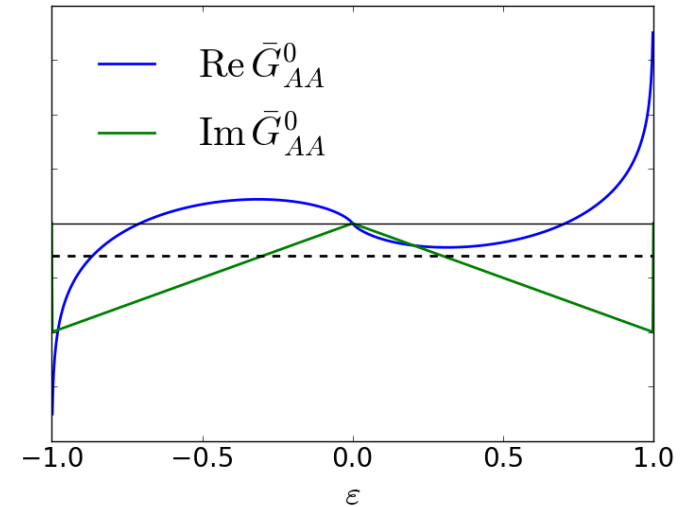
# Resonant impurity

Impurity  $T$ -matrix:

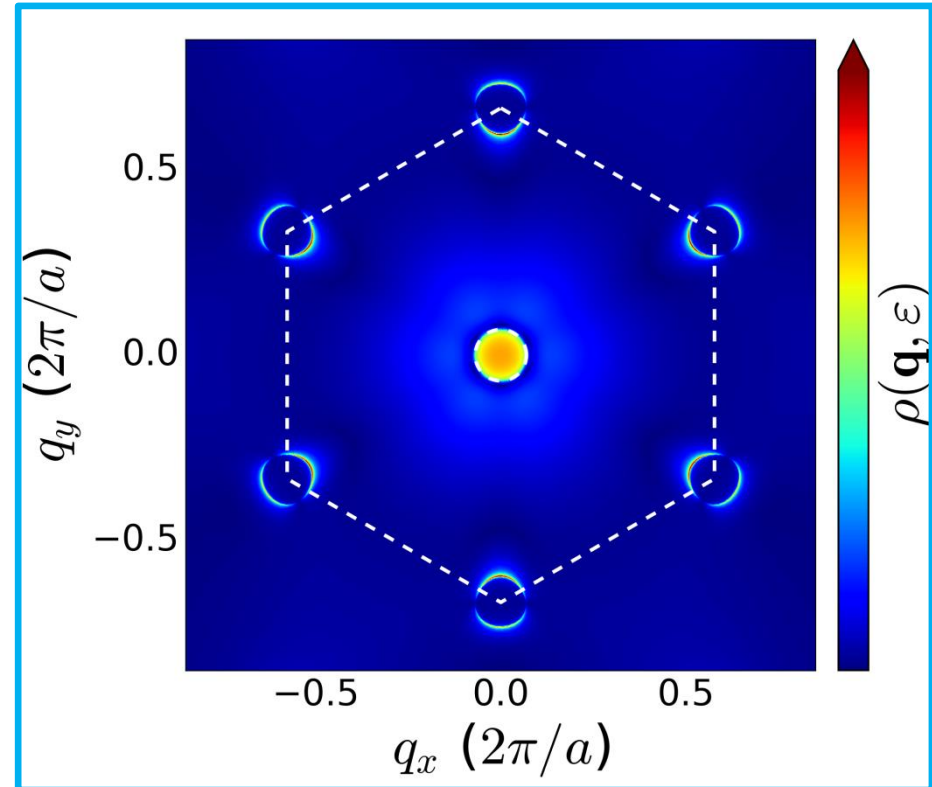
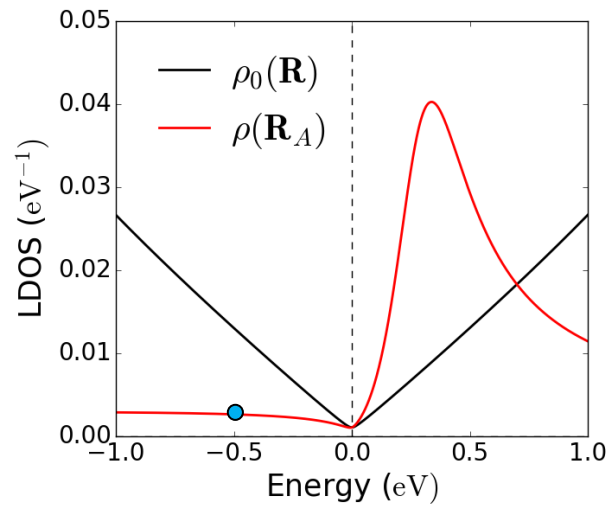
$$T_{AA}(\varepsilon) = V_0 / [1 - V_0 \bar{G}_{AA}^0(\varepsilon)]$$

$$\bar{G}_{ij}^0(\varepsilon) = \frac{1}{N} \sum_{\mathbf{k}} G_{\mathbf{k}}^{0,ij}(\varepsilon)$$

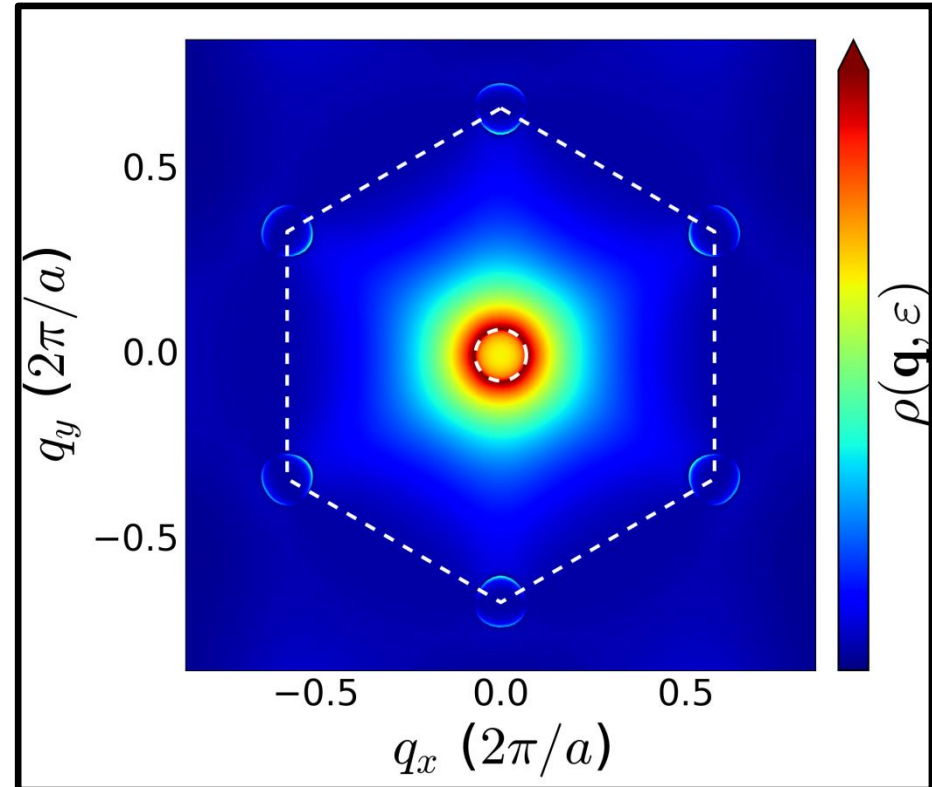
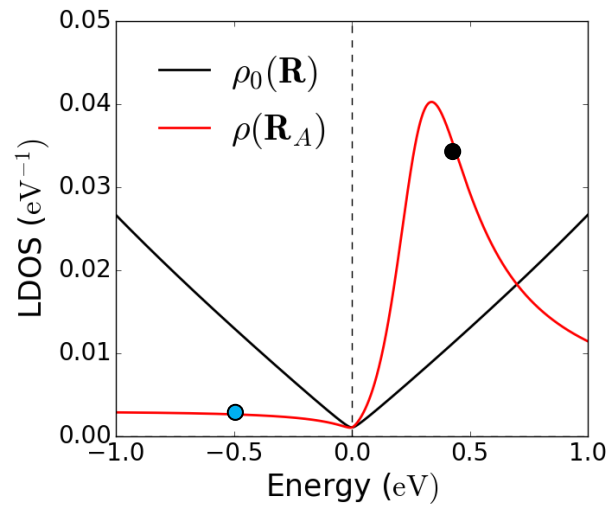
Strong impurity:  $V_0 = -16 \text{ eV}$



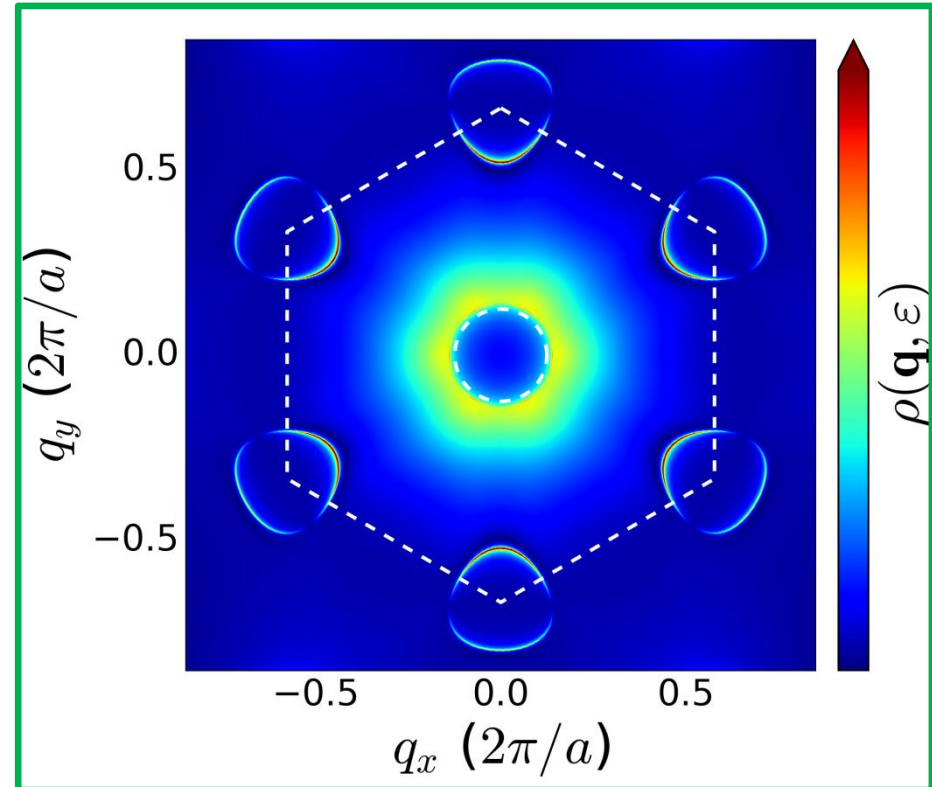
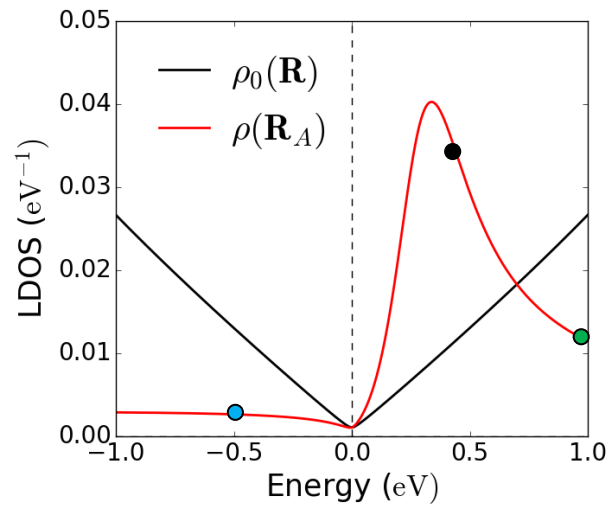
# FT-STIS for resonant scattering



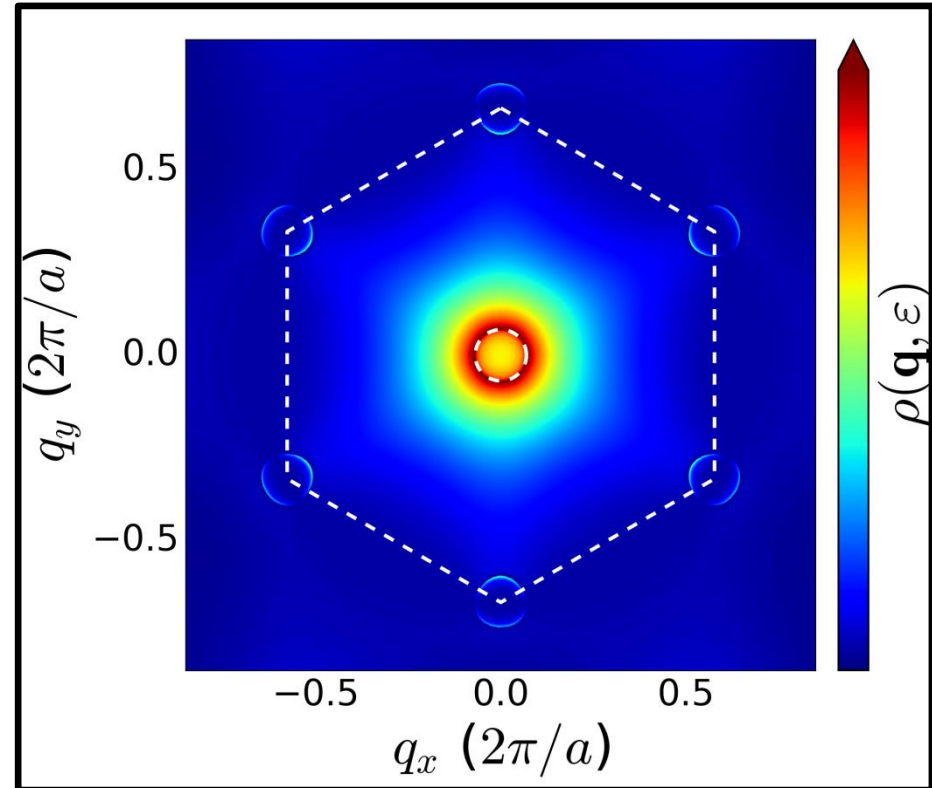
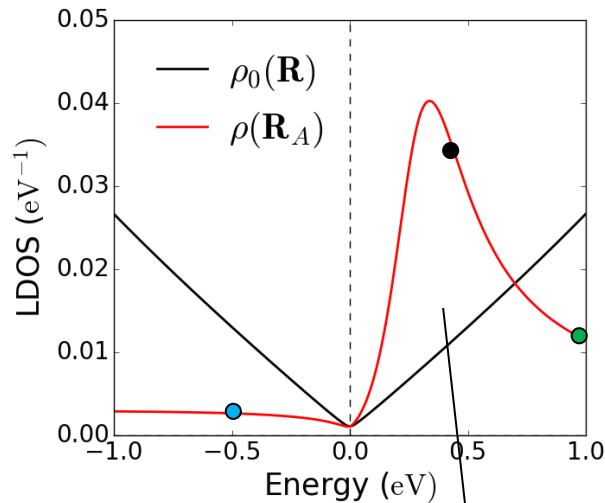
# FT-STS for resonant scattering



# FT-STS for resonant scattering



# FT-STs for resonant scattering

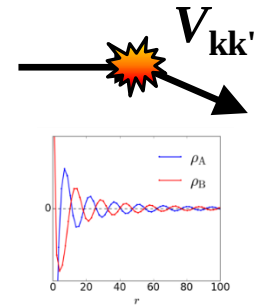


$$|\Psi_{s\mathbf{k}}(\varepsilon)\rangle = |\psi_{s\mathbf{k}}\rangle + \sum_{\mathbf{k}'} |\psi_{s\mathbf{k}'}\rangle \hat{G}_{\mathbf{k}'}^0(\varepsilon) \hat{T}_{\mathbf{k}'\mathbf{k}}(\varepsilon)$$

- Reappearance of broad backscatter ring at resonance  
→ standard “scattering picture” breaks down.

# Summary

1. QPI/FT-STs unique method for probing QP states and the scattering properties of defects → direct fingerprint.
2. Unified theoretical framework for calculating FT-STs/QPI spectra in two-dimensional materials:
  - Impurity matrix element (scattering processes).
  - Phase-factor matrix element (interference).
3. The latter can mask scattering processes in FT-STs:
  - intravalley backscattering in graphene (spinor overlap).
4. Uncovered a hitherto unexplored variant of the backscatter ring for resonant scattering in graphene.
5. Relevant for QPI in Moiré lattices, other 2D materials (e.g., silicene), as well as surfaces of TIs and Weyl semimetals.



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