Graphene 2018 – June 26-29 (Dresden, Germany)

Two component superfluid Bose-Einstein condensate of indirect excitons in two-layer Hall systems complementary filled

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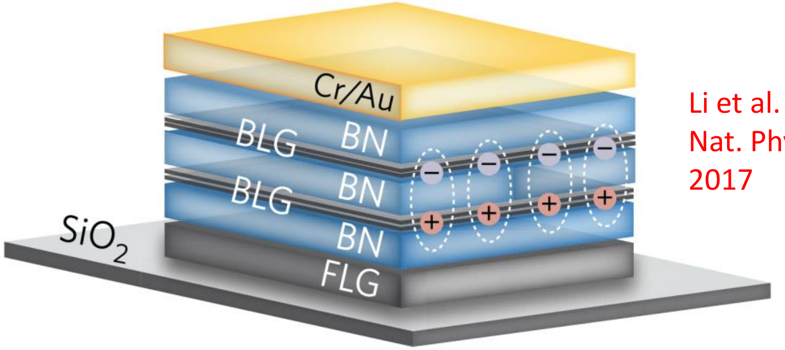
- Twin Hall systems
- Thin barrier blocking inter-layer tunneling
- hBN for graphene, GaAlAs for GaAs

$$d \propto l_B, \quad l_B = \sqrt{\frac{\hbar}{eB}}$$

~3nm graphene
~10nm GaAs

superfluidity of Bose-Einstain condensate of indirect excitons
in competition with
reentrant IQHE



Li et al. Nat. Phys. 2017

$$\nu_T = \nu_{\text{top}} + \nu_{\text{bot}} = 1$$

Theory:
- Lozovik 1976
- Gorkov et al. 1968
- Halperin et al. 1984
- Fertig 1987
- Mac Donald et al. 2001
- Jacak 2018

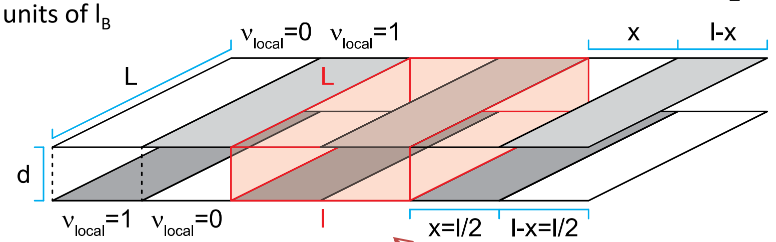
Experiment:
- b-graphene/hBN/b-graphene
  - Kim et al. 2017
  - Dean et al. 2017
- GaAs/GaAlAs/GaAs
  - Klitzing et al. 2004
  - Eisenstein et al. 2003
  - Tutuc et al. 2007

our proposition: the stripe-model

stripes with ν=1 alternately with ν=0 complementary in both layers

all dimensions in units of l_B

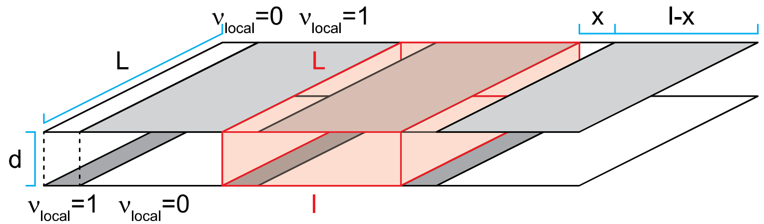
$$\nu_T = \nu_{\text{top}} + \nu_{\text{bot}} = 1$$



balanced filling

$x=l/2 \rightarrow \nu_{top} = \nu_{bot} = 1/2$

$\nu_{top}=(l-x)/l$, $\nu_{bot}=x/l$



imbalanced filling

$x < l/2 \rightarrow \nu_{top}=(l-x)/l > \nu_{bot}=x/l$

energy minimization with Metropolis Monte Carlo sim.

4-stripe sector selected for energy minimization

$$\Delta E = \boxed{\Delta E_{top}} + \boxed{\Delta E_{bot}} + \boxed{\Delta E_{inter}}$$

$$\begin{aligned}\Delta E_{top} =& \mathrm{el}_{top}\left[\nu_{local}=1\right] \leftrightarrow \mathrm{jell}_{top}\left[\rho=\tfrac{l-x}{l2\pi}\right]\\ &-\mathrm{el}_{top}\left[\nu_{nom}=\tfrac{l-x}{l}\right] \leftrightarrow \mathrm{jell}_{top}\left[\rho=\tfrac{l-x}{l2\pi}\right]\\ &+\mathrm{el}_{top}\left[\nu_{local}=1\right] \leftrightarrow \mathrm{el}_{top}\left[\nu_{local}=1\right]\\ &-\mathrm{el}_{top}\left[\nu_{nom}=\tfrac{l-x}{l}\right] \leftrightarrow \mathrm{el}_{top}\left[\nu_{nom}=\tfrac{l-x}{l}\right]\end{aligned}$$

$$\begin{aligned}\Delta E_{bot} =& \mathrm{el}_{bot}\left[\nu_{local}=1\right] \leftrightarrow \mathrm{jell}_{bot}\left[\rho=\tfrac{x}{l2\pi}\right]\\ &-\mathrm{el}_{bot}\left[\nu_{nom}=\tfrac{x}{l}\right] \leftrightarrow \mathrm{jell}_{bot}\left[\rho=\tfrac{x}{l2\pi}\right]\\ &+\mathrm{el}_{bot}\left[\nu_{local}=1\right] \leftrightarrow \mathrm{el}_{bot}\left[\nu_{local}=1\right]\\ &-\mathrm{el}_{bot}\left[\nu_{nom}=\tfrac{x}{l}\right] \leftrightarrow \mathrm{el}_{bot}\left[\nu_{nom}=\tfrac{x}{l}\right]\end{aligned}$$

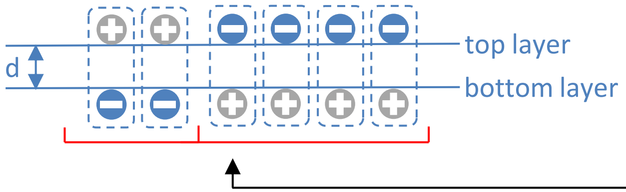
$$\begin{aligned}\Delta E_{inter} =& \mathrm{el}_{top}\left[\nu_{local}=1\right] \leftrightarrow \mathrm{jell}_{bot}\left[\rho=\tfrac{x}{l2\pi}\right]\\ &-\mathrm{el}_{top}\left[\nu_{nom}=\tfrac{l-x}{l}\right] \leftrightarrow \mathrm{jell}_{bot}\left[\rho=\tfrac{x}{l2\pi}\right]\\ &+\mathrm{el}_{bot}\left[\nu_{local}=1\right] \leftrightarrow \mathrm{jell}_{top}\left[\rho=\tfrac{l-x}{l2\pi}\right]\\ &-\mathrm{el}_{bot}\left[\nu_{nom}=\tfrac{x}{l}\right] \leftrightarrow \mathrm{jell}_{top}\left[\rho=\tfrac{l-x}{l2\pi}\right]\\ &+\mathrm{el}_{top}\left[\nu_{local}=1\right] \leftrightarrow \mathrm{el}_{bot}\left[\nu_{local}=1\right]\\ &-\mathrm{el}_{top}\left[\nu_{nom}=\tfrac{l-x}{l}\right] \leftrightarrow \mathrm{el}_{bot}\left[\nu_{nom}=\tfrac{x}{l}\right]\end{aligned}$$

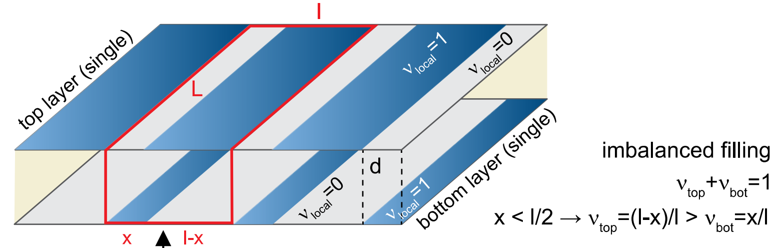
Reentrant IQHE el-el – Metropolis Monte Carlo

energies from Metropolis MC sim. for ν = 1 IQHE state, in units $\frac{e^2}{l_B}\frac{1}{4\pi\varepsilon_0}$

N	$E_{\text{jell-jell}}/N$	$E_{\text{jell-el}}/N$	$E_{\text{el-el}}/N$	$\Delta E_{\text{total}}/N$
80	5.36845	-10.6326	4.6936	-0.570516
160	7.59213	-15.1761	6.95414	-0.629868
240	9.29843	-18.5908	8.672	-0.620415
320	10.7369	-21.4734	10.1046	-0.631899
400	12.0042	-24.0026	11.3821	-0.616286
480	13.15	-26.2682	12.4993	-0.618932

indirect excitons – electrons and holes located in opposite layers

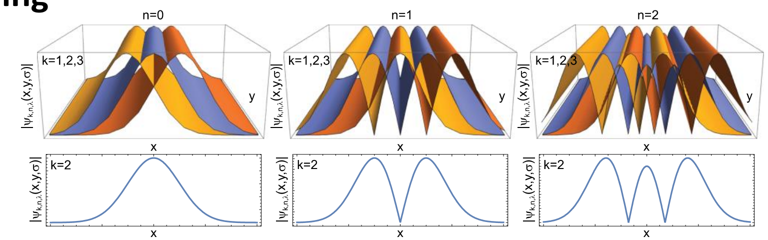




wave-function of indirect excitons in Landau gauge → **k-space striping**

$$\psi_{k,n,\lambda}(x,y,\sigma)$$
$$= Ce^{iky-\frac{(x+\alpha k l_B^2)}{2l_B^2}}\omega_n\left(\frac{x+\alpha k l_B^2}{l_B^2}\right)\delta_{\lambda,\sigma}$$
$$\lambda,\sigma = 1,2$$

LLL (n=0),
α=1 for electron
α=-1 for hole
w_n – Hermite polynomial
the same density for *αk=const*



two-component Bogolubov model (BEC superfluidity)

$$H = H^a + H^b = \sum_p E_p^a a_p^+ a_p + \frac{1}{2S}\sum_{\substack{p_1,p_2,p_3,p_4\\ p_1+p_2=p_3+p_4}} u_a(|p_1-p_4|)a_{p_1}^+ a_{p_2}^+ a_{p_3} a_{p_4} + \sum_p E_p^b b_p^+ b_p + \frac{1}{2S}\sum_{\substack{p_1,p_2,p_3,p_4\\ p_1+p_2=p_3+p_4}} u_b(|p_1-p_4|)b_{p_1}^+ b_{p_2}^+ b_{p_3} b_{p_4}$$

$$a_0^+ = a_0 = \sqrt{N_0^a}$$

$$H^a \simeq \frac{1}{2S}u(0)N_0^{a2} + \sum_p\left[E_p^a + \frac{N_0^a}{S}u_a(|p|)\right]a_p^+ a_p + \frac{N_0^a}{2S}\sum_{p\neq 0} u_a(|p|)(a_p^+ a_{-p}^+ + a_p a_{-p})$$

Bogolubov diagonalization

$$a_p = \alpha(p)\tilde{a}_p + \beta^*(p)\tilde{a}_{-p}^+,\ a_p^+ = \alpha^*(p)\tilde{a}_p^+ + \beta(-p)\tilde{a}_{-p}$$

$$H^a \simeq \frac{1}{2S}u_a(0)N_0^{a2} + \Delta E^a + \sum_p \epsilon_p^a \tilde{a}_p^+ \tilde{a}_p$$

superfluidity condition

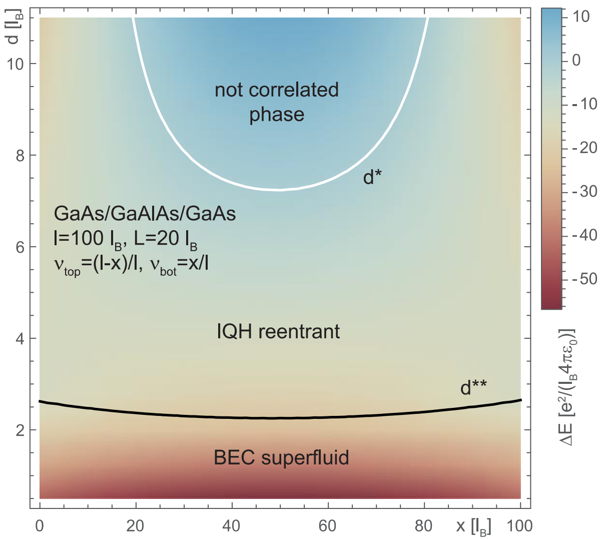
$$\boxed{\epsilon_p^a \simeq c^a|p|} \quad c^a = \sqrt{\frac{N_0^a u_a(0)}{m_a^* S}}$$

excitons repulsion

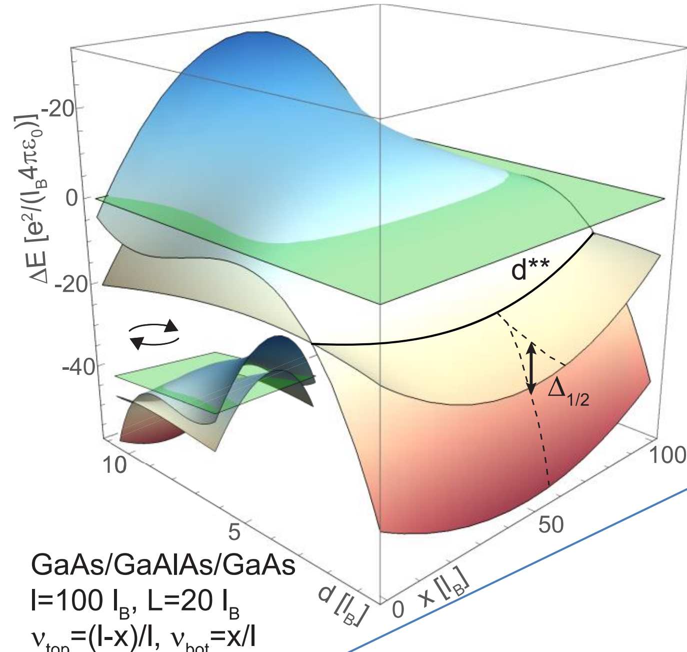
$$\boxed{u_a(0) > 0}$$

- the indirect exciton system has **two-component** → <u>two possible exciton vertical polarizations</u>
- for repulsion optimization the stripe structure is also necessary → stripes in k-space, which is equivalent with stripes in both layers with local filling *v = 1* alternately with *v = 0*

- energy competition between different phases → phase diagrams
- energies estimated with the MMC simulation

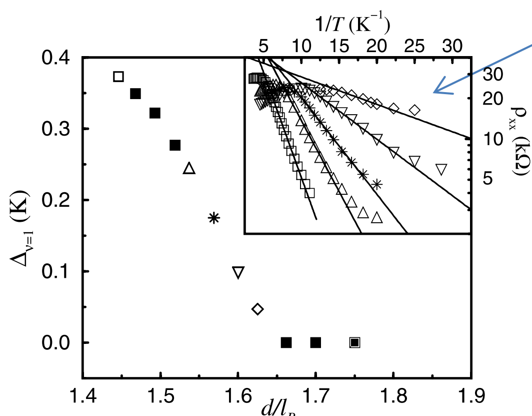


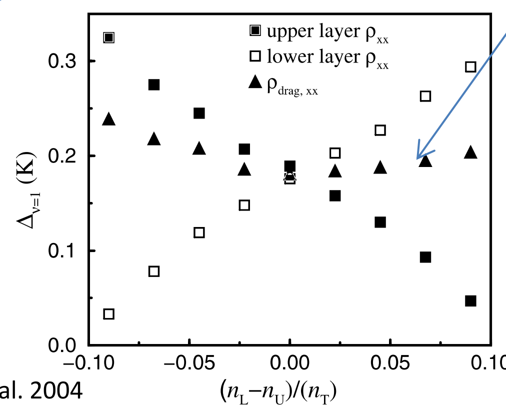
Jacak 2018



- 3d view of hyperplanes of competing energies
- for the balanced filling the activation energy $\Delta_{1/2}$ for superfluid phase compared with experimental value (taken from the temperature dependence of the resistivity)
- the concave shape of the critical curve d** in agreement with the measured imbalance dependence of the drag longitudinal resistivity

Klitzing et al. 2004

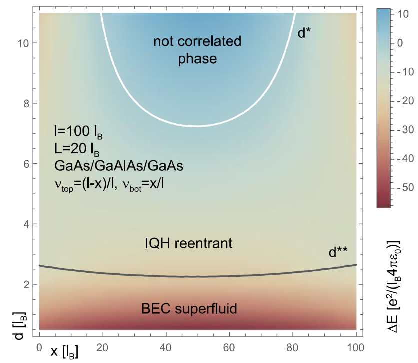


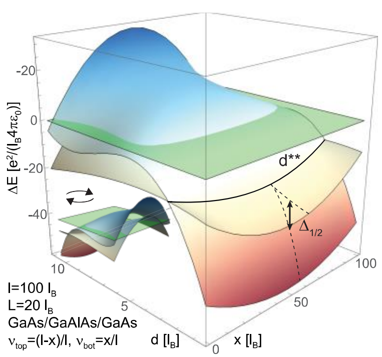


Klitzing et al. 2004

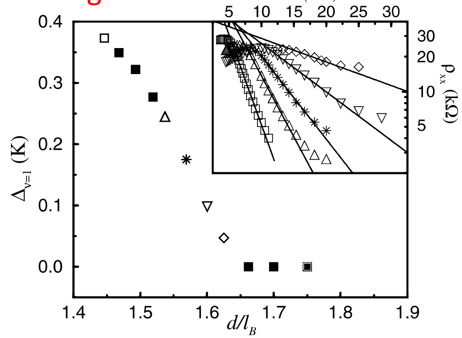
comparison with the experimental data

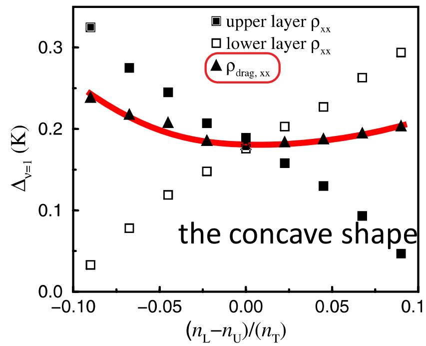
Jacak 2018



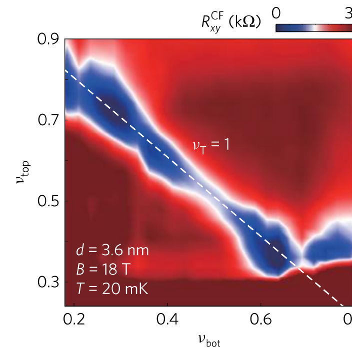


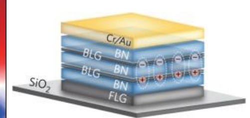
Klitzing et al. 2004

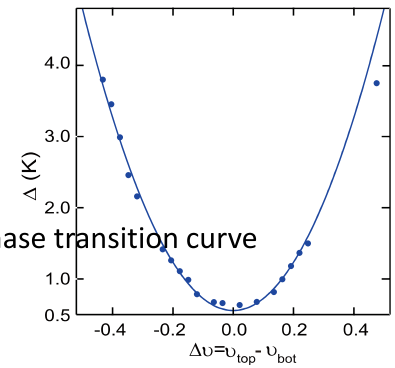




Kim et al. 2017
bg/hBN/bg

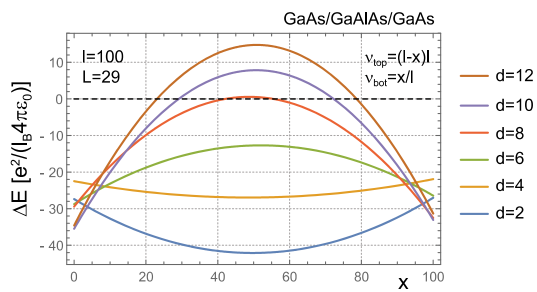




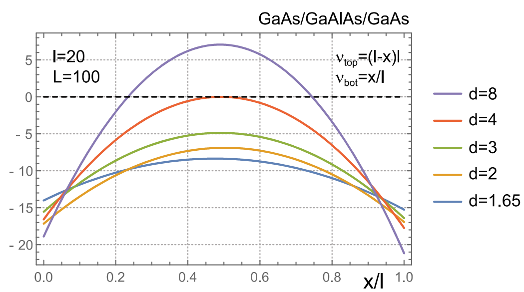


the concave shape of the phase transition curve

the evolution of the hyperplane (concave ↔ convex) profile for IQHE reentrant phase, depending on d for GaAs/GaAlAs/GaAs
l/L ~ 4 (optimal)



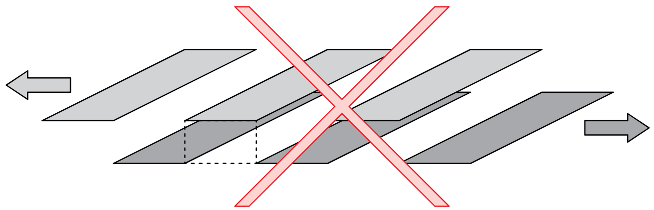
optimal stripe geometry

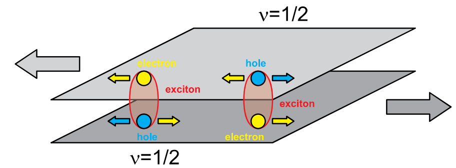


suboptimal stripe geometry

two disitnct dymanic configurations

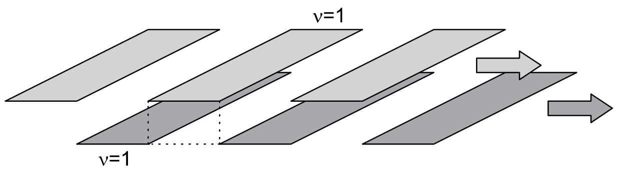
`counterflow` configuration
(layers connected on one end)
→ current flows in oposite directions in layers



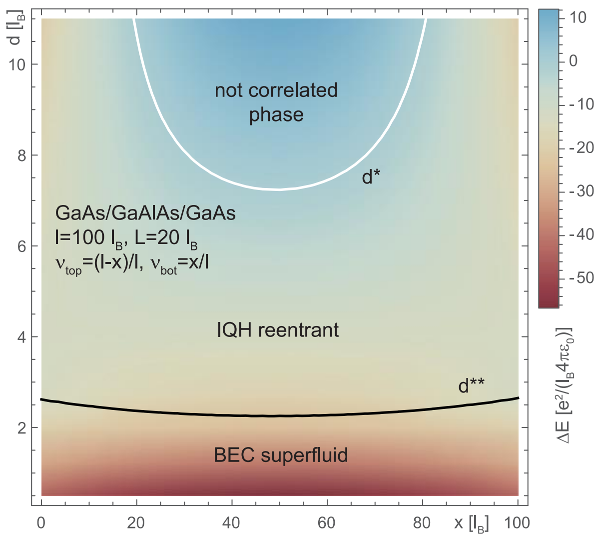


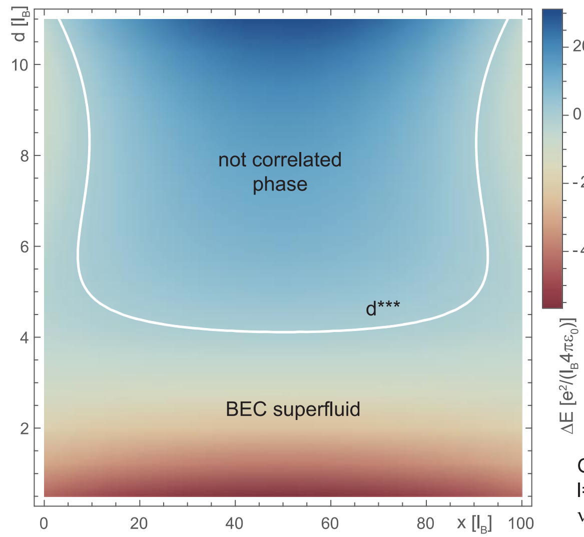
BCS superfluidity
→ not IQHE

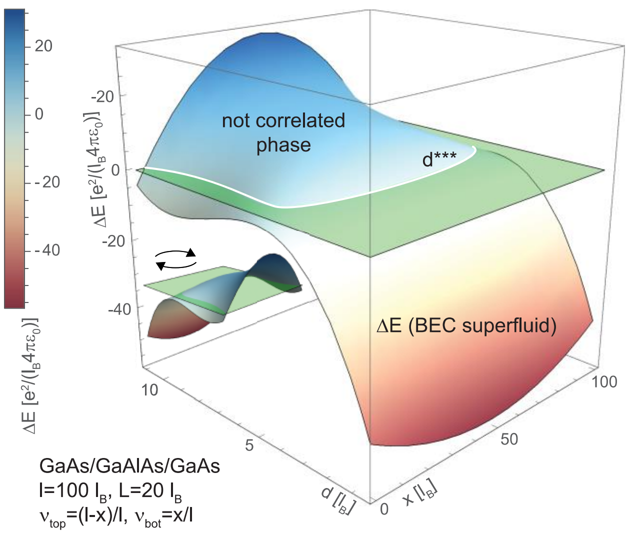
`parallel drag` configuration
→ current in one layer drags the carriers in the other one



stripe structure maintains → IQHE



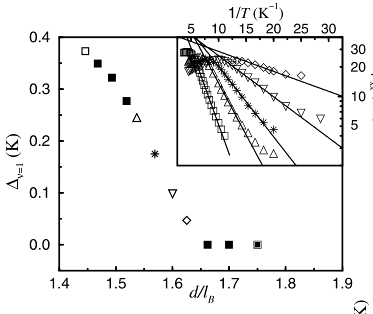




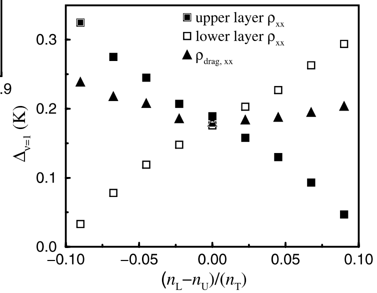
phase diagram for counterflow configuration

disappearance of the reentrant IQHE phase → the domination of the superfluid BEC phase up to curve d*** (0 energy gain)

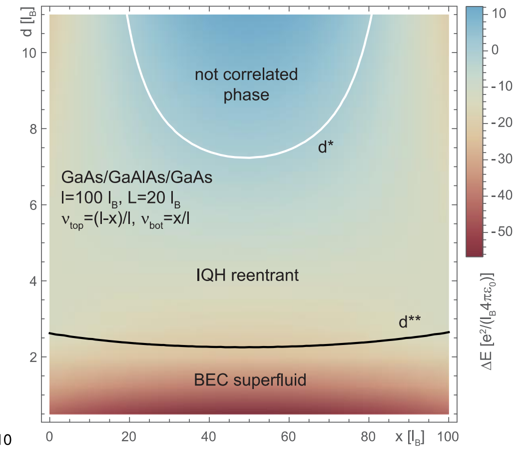
comparison with the experimental data

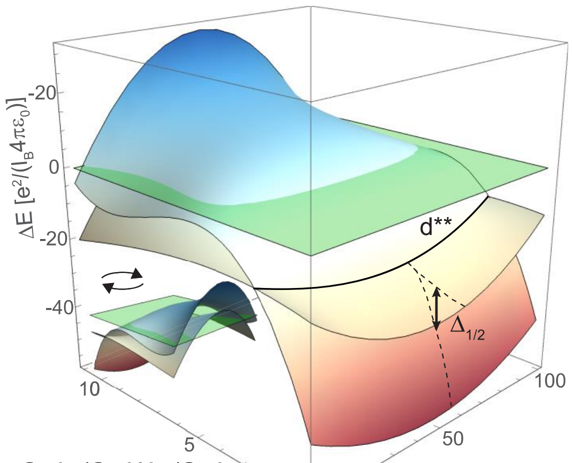


Klitzing et al. 2004

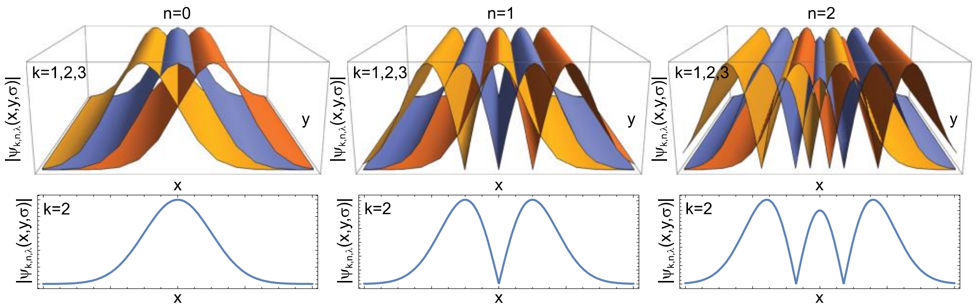


Jacak 2018





$$\psi_{k,n,\lambda}(x,y,\sigma) = Ce^{iky-(x+\alpha k l_B^2)^2/2l_B^2} w_n\left(\frac{x+\alpha k l_B^2}{l_B}\right)\delta_{\lambda,\sigma}, \quad \lambda,\sigma = 1,2$$



- → wavefunction profiles for k = 1,2,3 in LL with n = 0,1,2 (from left) in the Landau gauge A = (0, Bx, 0))
- → **for n = 1 lack of the central maximum**
- → **for n = 0 or 2 the central maximum is present**
- → resulting in ca. 4 times lower interaction of the nearest and next-nearest electrons in the n = 1 subband in comparison to the n = 0 or 2
- → leading to the absence of the superfluidity of the BEC condensate for n = 1 LLL in b-graphene (visible in exp. Liu et al. Nat. Phys. 2017, Li et al. Nat. Phys. 2017)

lack of the Hall response for: $\left(-\frac{1}{2},-\frac{1}{2}\right)$

- → in the hole stripe there is only bare jellium – fully filled valence band with the gap (in b-graphene) to the conductance band → Hall response inactive
- → otherwise in (-3/2,-1/2) with additional hole degrees of freedom

Hall response fully developed for: $\left(\frac{1}{2},\frac{1}{2}\right),\left(-\frac{3}{2},-\frac{1}{2}\right)$