



MINISTERIO  
DE ECONOMÍA  
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# Superlensing with twisted bilayer graphene

Tobias Stauber and Heinrich Kohler

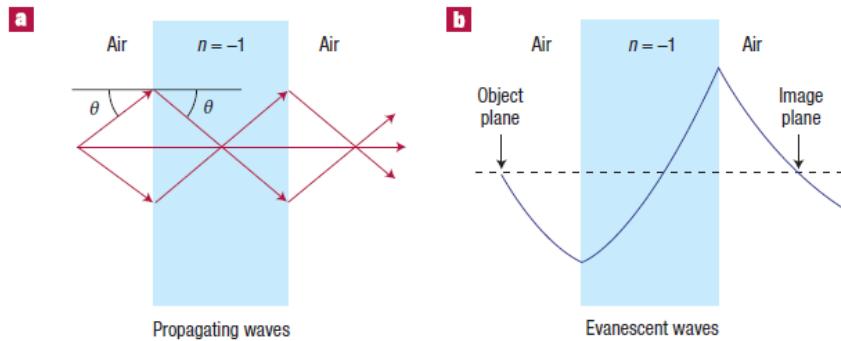
Instituto de Ciencia de Materiales de Madrid, CSIC

Graphene 2017 - Barcelona, 29/03/2017

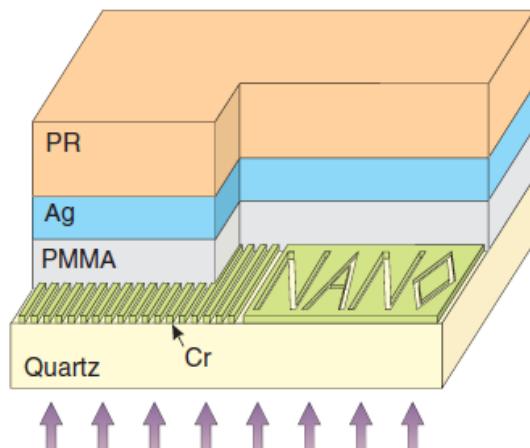
# Introduction

## Perfect lensing and hyperlensing

Planar superlenses reconstitute the near-field of the source by virtue of resonant surface waves.

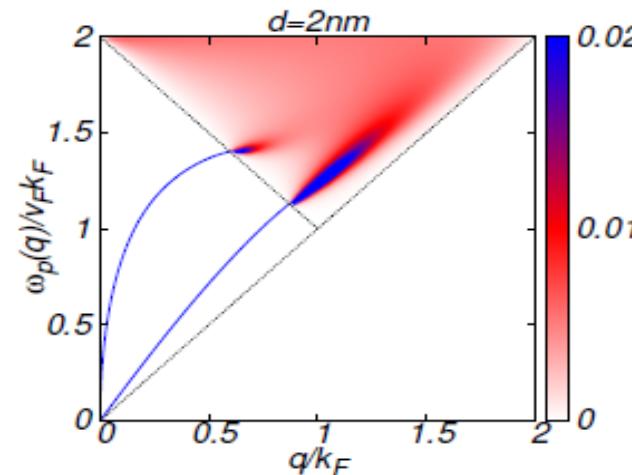
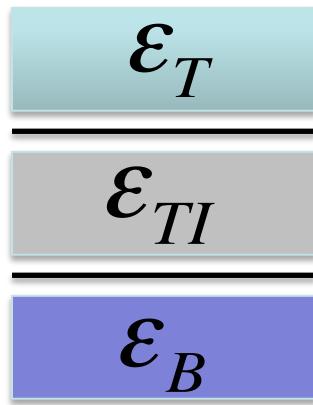


J. B. Pendry, Phys. Rev. Lett. 85, 3966 (2000)



N. Fang et al., Science 308, 534 (2005)

## Plasmons in double layer systems, e.g., Topological Insulators



Optical Mode

$$\omega_+^2 = \frac{\alpha v_F^2 (k_F^T + k_F^B)}{(\epsilon_T + \epsilon_B)} q$$

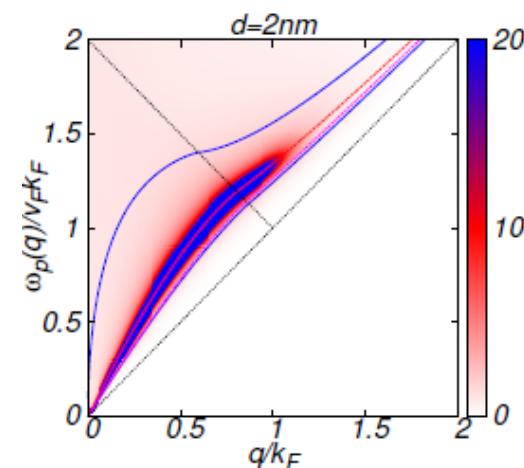
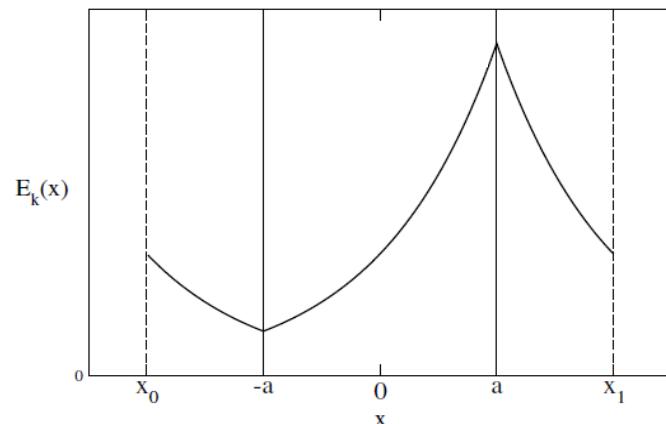
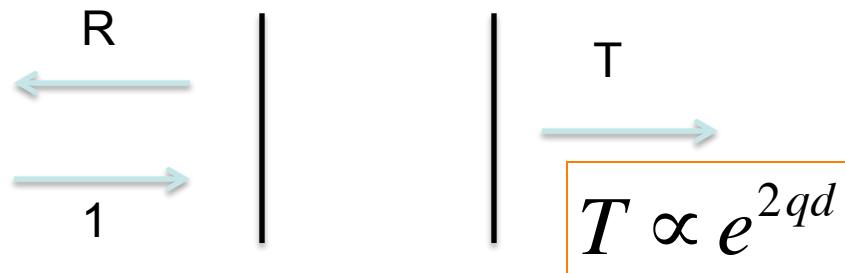
Acoustic Mode

$$\omega_-^2 = \frac{\alpha v_F^2 k_F^T k_F^B}{\epsilon_{TI} (k_F^T + k_F^B)} q^2$$

R. E. V. Profumo et al., Phys. Rev. B 85, 085443 (2012)

## Exponential amplification of evanescent modes

Exponential amplification of evanescent modes for  $R=0$ .



**WANTED:**

*Plasmonic mode with constant energy dispersion*

Analogy to Pendry's perfect lens

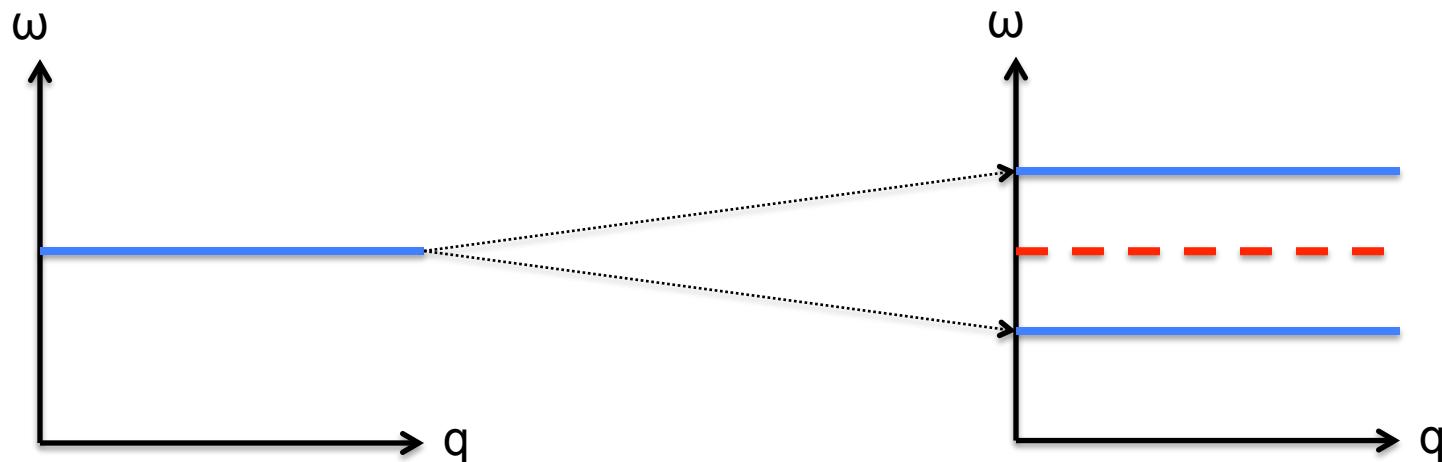
T. Stauber and G. Gómez-Santos, Phys. Rev. B 85, 075410 (2012)

## Exponential amplification of evanescent modes

1 layer



2 layers



Exponential amplification for all  
modes at constant energy

## Hydrodynamic models for 2D systems

Continuity equation and linear response yields:

$$\omega^2 = \chi(\omega, q) \frac{e^2 q}{2\epsilon_0 \kappa}$$

Approximate current response by Drude weight D:

$$D = e^2 \chi(\omega \rightarrow 0, q = 0)$$

Plasmon dispersion in local approximation for 2D systems:

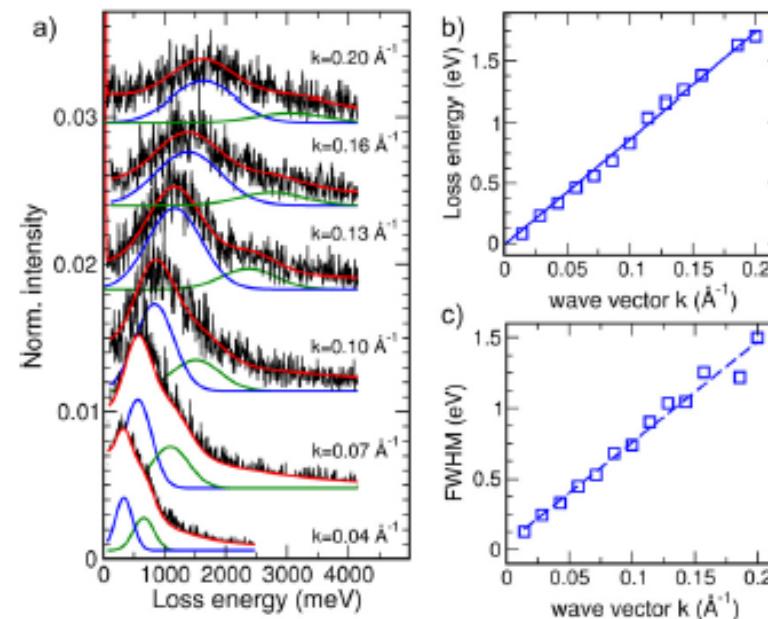
$$\omega_p = \sqrt{\frac{D}{2\epsilon_0 \kappa}} q$$

## Interband “plasmons” in Graphene on Ir(111)

Do plasmons exist in a neutral system ( $D=0$ )

EELS of Graphene on Ir

Ir



T. Langer et al., New J. Phys. **13**, 053006 (2011)

## Interband “plasmons” in Graphene on Ir(111)

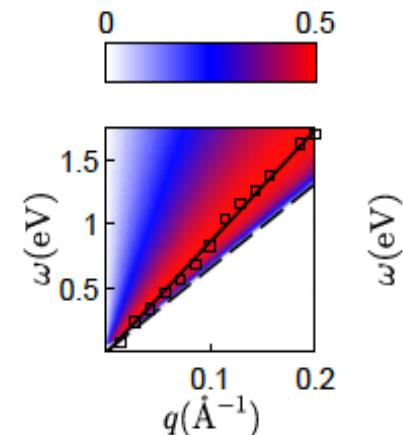
In pure Dirac systems, there are no interband plasmons because the charge response is always negative.

$$\chi_\rho(q, \omega) = -\frac{1}{4\hbar} \frac{q^2}{\sqrt{(v_F q)^2 - \omega^2}} < 0$$

$$\omega^2 \neq \chi(q, \omega) \frac{e^2 q}{2\epsilon_0 K}$$

But there can be an enhanced charge response as seen in the maximum of the loss function.

$$S(q, \omega) = -\text{Im} \frac{1}{1 - v_q \chi_\rho(q, \omega)}$$



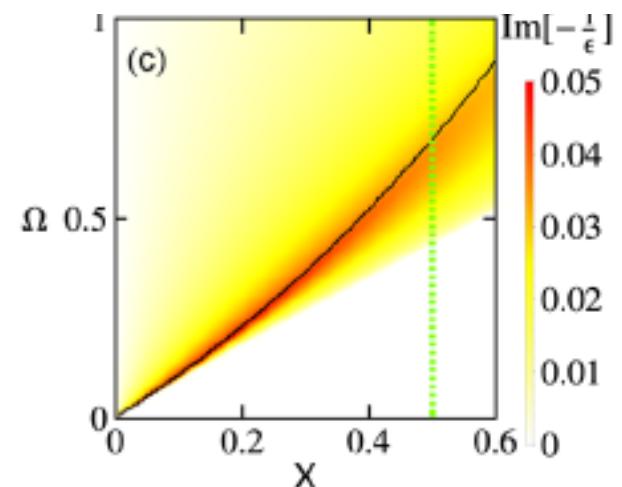
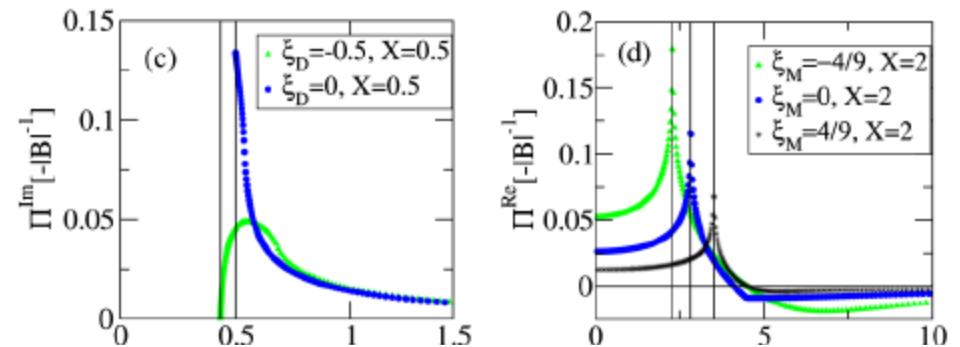
## Are interband plasmons possible?

BHZ-model for fermions in  
Hg(Cd)Te:

$$H = \mathbf{d}_k \cdot \vec{\sigma}$$

$$\mathbf{d}_k = (v_F k_x, v_F k_y, M - B k^2)$$

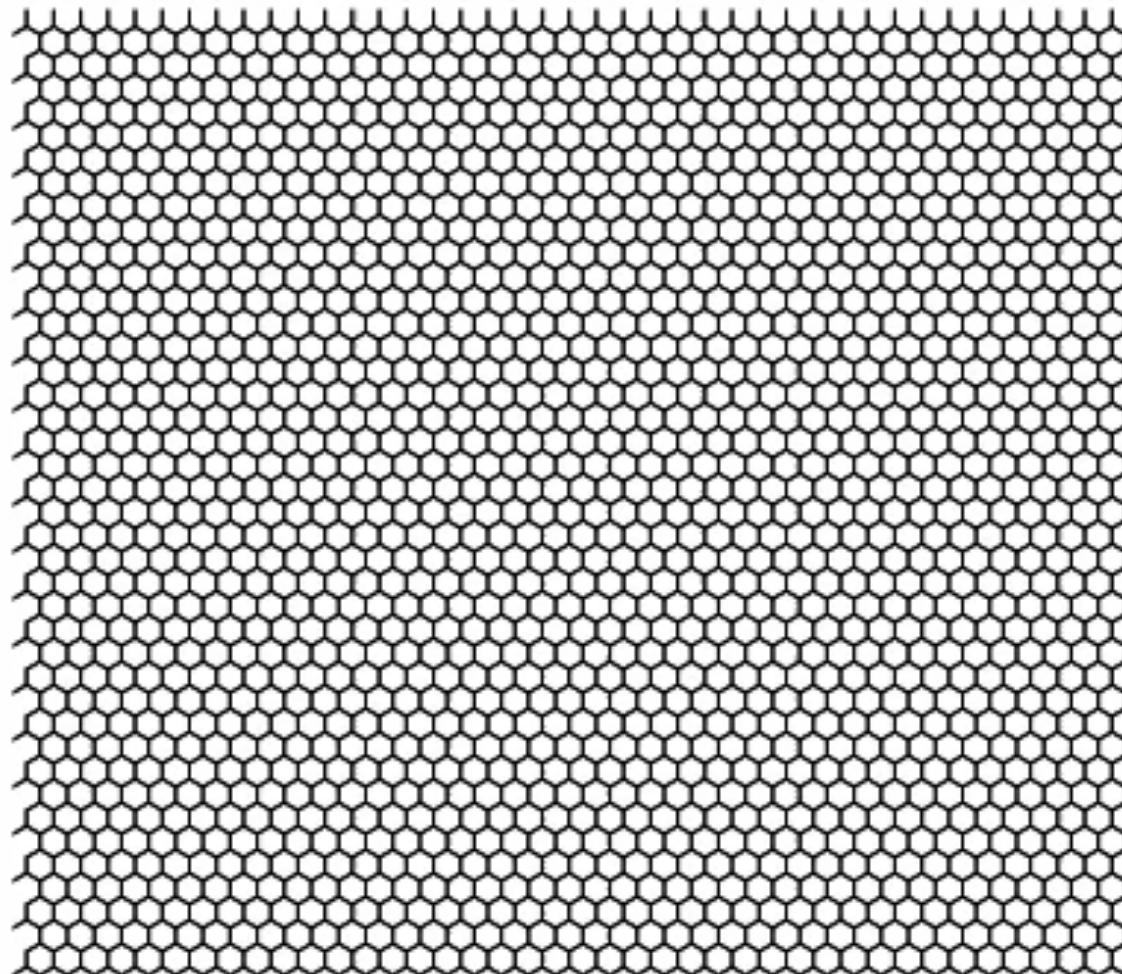
*Mixture between Dirac and Schrödinger electrons yields plasmons at zero doping*



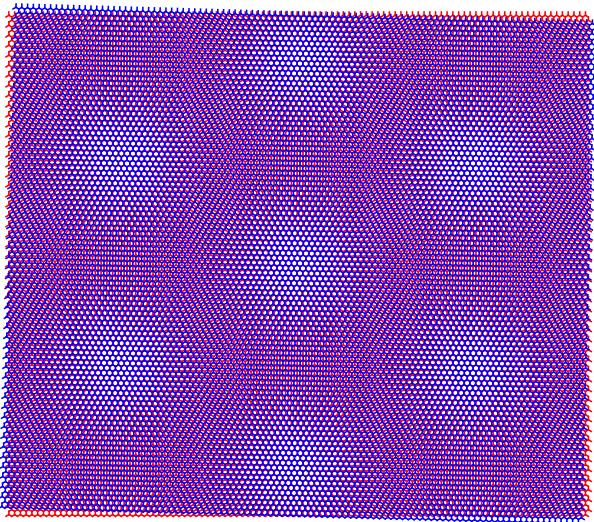
S. Juergens, P. Michetti, and B. Trauzettel, Phys. Rev. Lett. 112, 076804 (2014)

# Twisted bilayer graphene

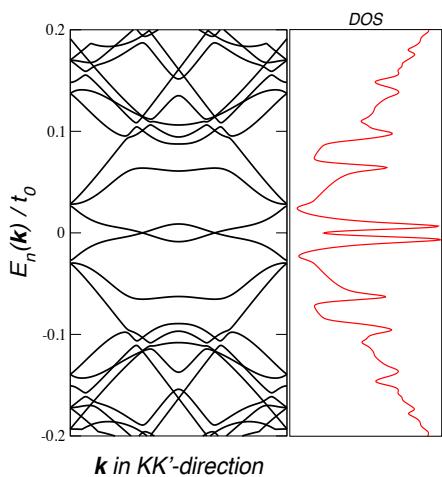
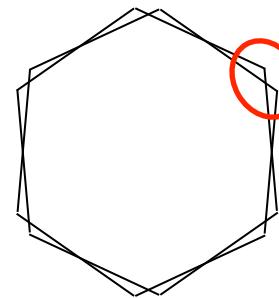
## Twisted bilayer graphene



## Brillouin zone for twisted bilayer graphene



Two Dirac cones

Twist angle parameterized by  $i$ 

$$\cos \theta_i = 1 - \frac{1}{2A_i}$$

$$A_i = 3i^2 + 3i + 1$$

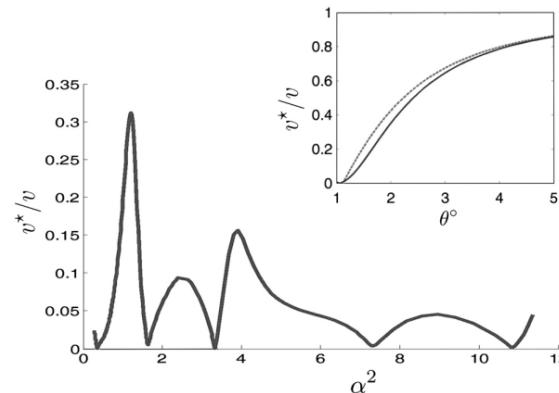
## Fermi velocity renormalization

Renormalization of the Fermi velocity:

$$v = v_F \left( 1 - 9 \frac{t_\perp}{v_F \Delta K} \right)$$

J. M. B. Lopes dos Santos et al., Phys. Rev. Lett. **99**, 256802 (2007).

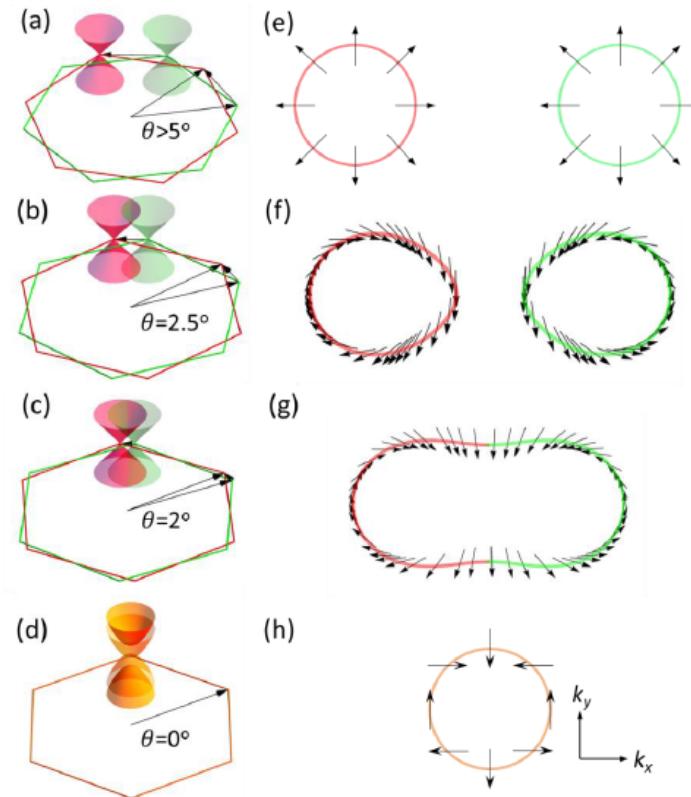
Appearance of magic angles for  $i > 31$



R. Bistritzer and A. H. MacDonald , PNAS **108**, 174108 (2011).

## Merging of the pseudo-spin texture

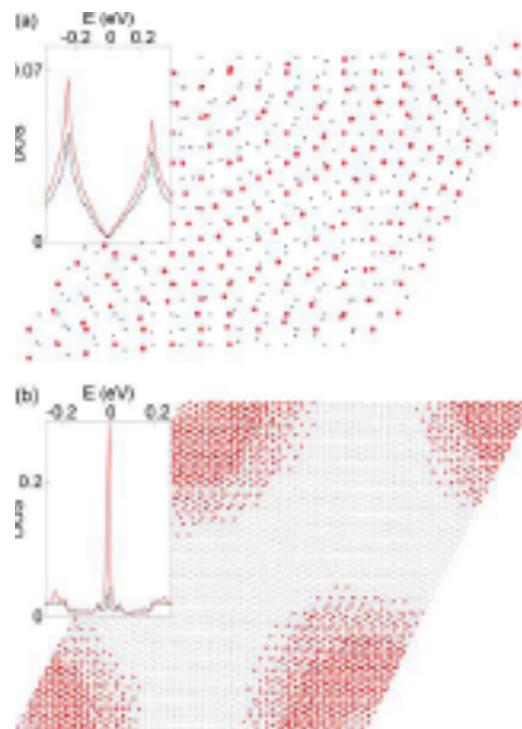
Crossover from large angle regime to low angle regime



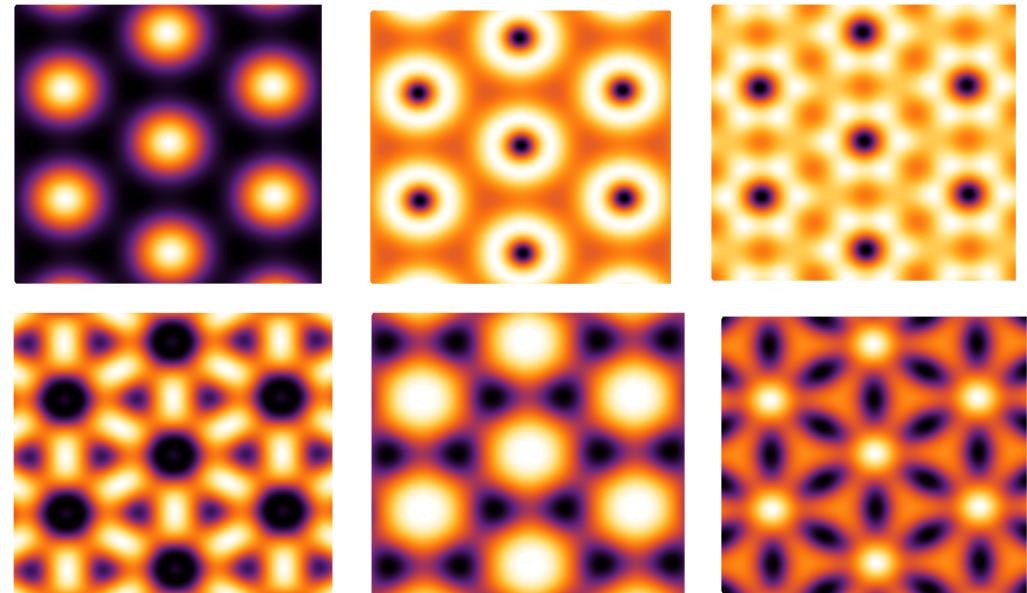
M. Zhu et al., 2D Mater. 4, 011013 (2017)

## Localized states around AA-stacked islands

## Crossover from extended to localized states



Local density of first six conduction bands:



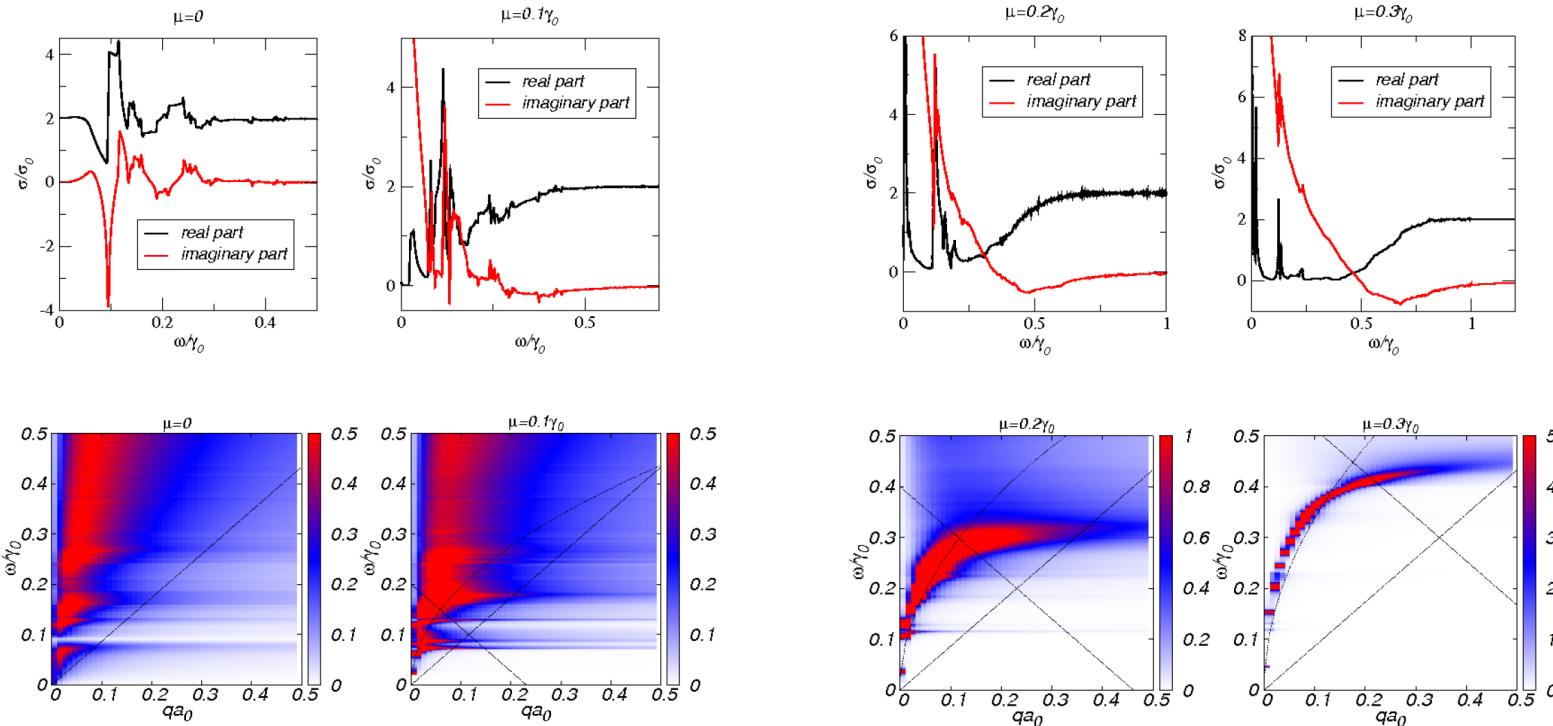
G. T. Trambly de Laissardiere, D. Mayou,  
L. Magaud, Nano Lett. 10, 804 (2010)

T. Stauber and H. Kohler, Nano Lett. **16**, 6844 (2016)

# Plasmons in twisted bilayer graphene

## Local optical response of twisted bilayer graphene

## Plasmons in local approximation

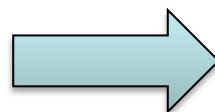


T. Stauber, P. San-Jose, and L. Brey, New J. Phys. , 804 (2013)

## Charge response with local field effects

Incoming momentum couples to reciprocal lattice vectors

$$V_{ext}(\mathbf{r}) = v_q e^{i\mathbf{q}\cdot\mathbf{r}}$$



$$\delta\rho(\mathbf{r}) = \sum_{\mathbf{G}} \delta\rho(\mathbf{q}, \mathbf{G}) e^{i(\mathbf{q}+\mathbf{G})\cdot\mathbf{r}}$$

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Charge susceptibility

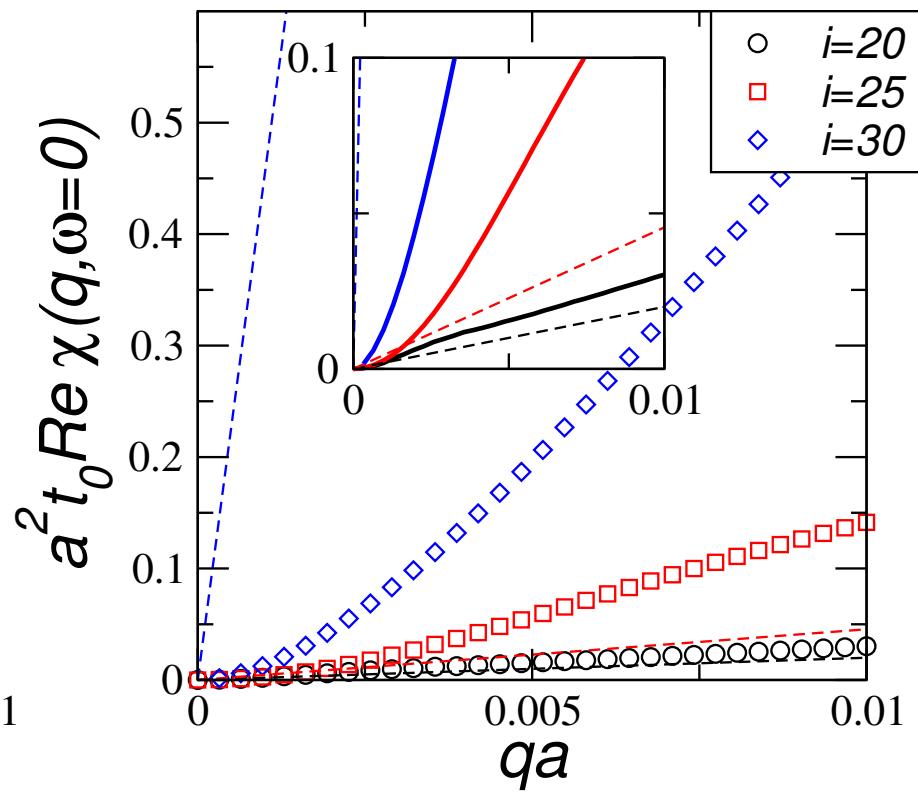
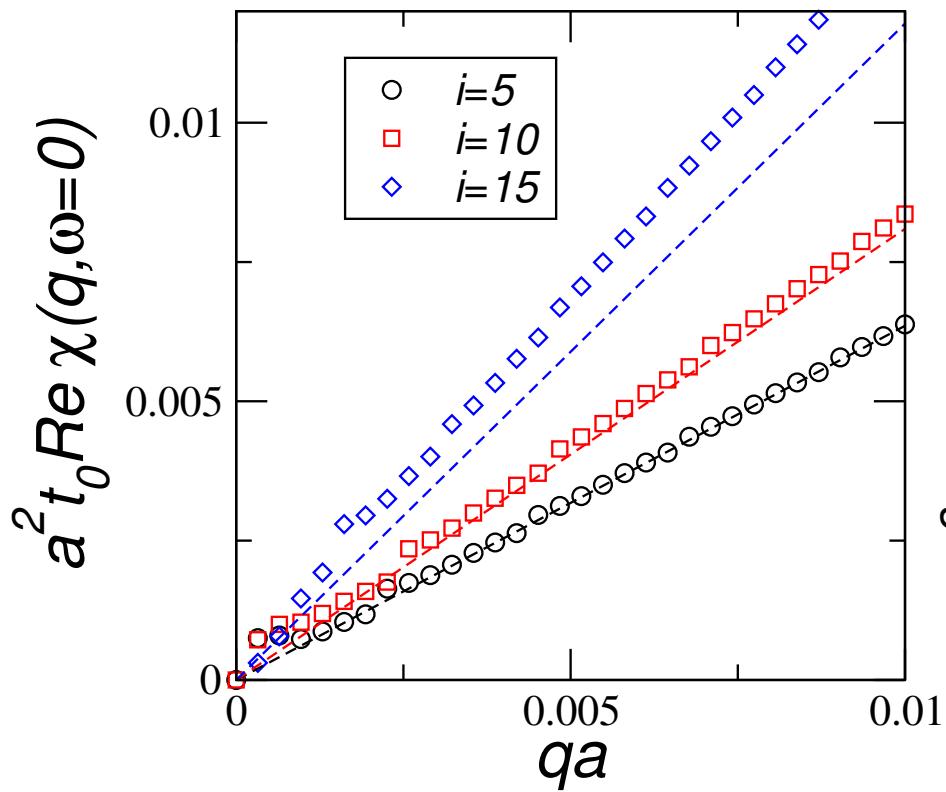
$$\chi_{\mathbf{G}, \mathbf{G}'}(\mathbf{q}, \omega) = \frac{g_s}{(2\pi)^2} \int_{1.BZ} d^2k \sum_{n,m;\kappa=\pm} f_{\mathbf{G}, \mathbf{G}'}^{n,m;\kappa}(\mathbf{k}, \mathbf{q}) \left[ \frac{n_F(E_{\mathbf{k}}^s) - n_F(E_{\mathbf{k}+\mathbf{q}}^{s'})}{E_{\mathbf{k}}^s - E_{\mathbf{k}+\mathbf{q}}^{s'} + \hbar\omega + i\delta} \right]$$

Band overlap including local field effects

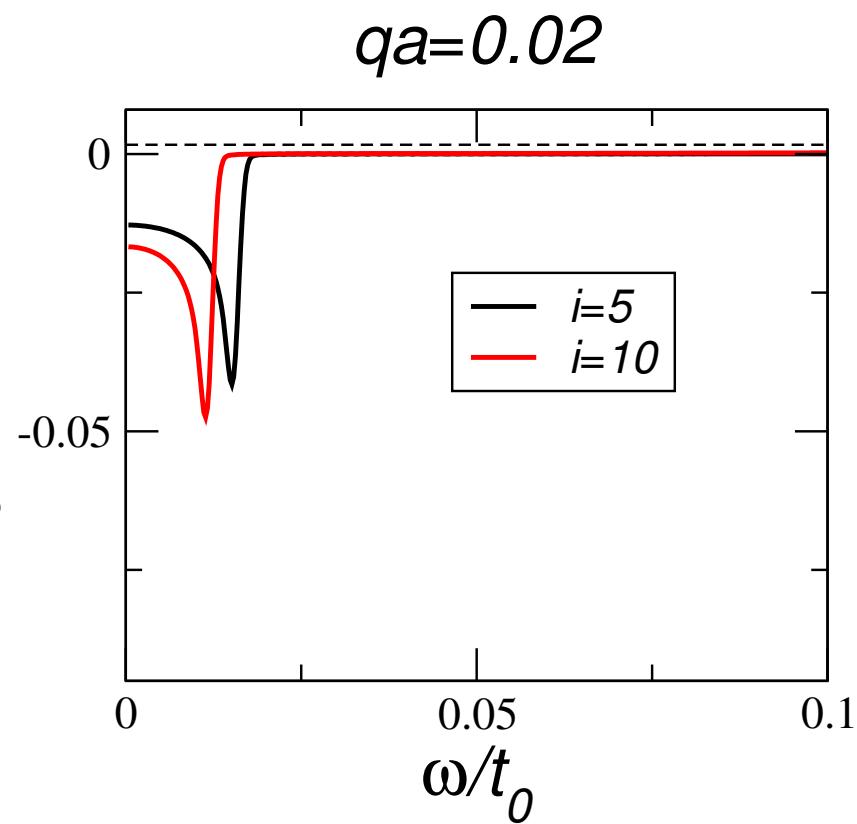
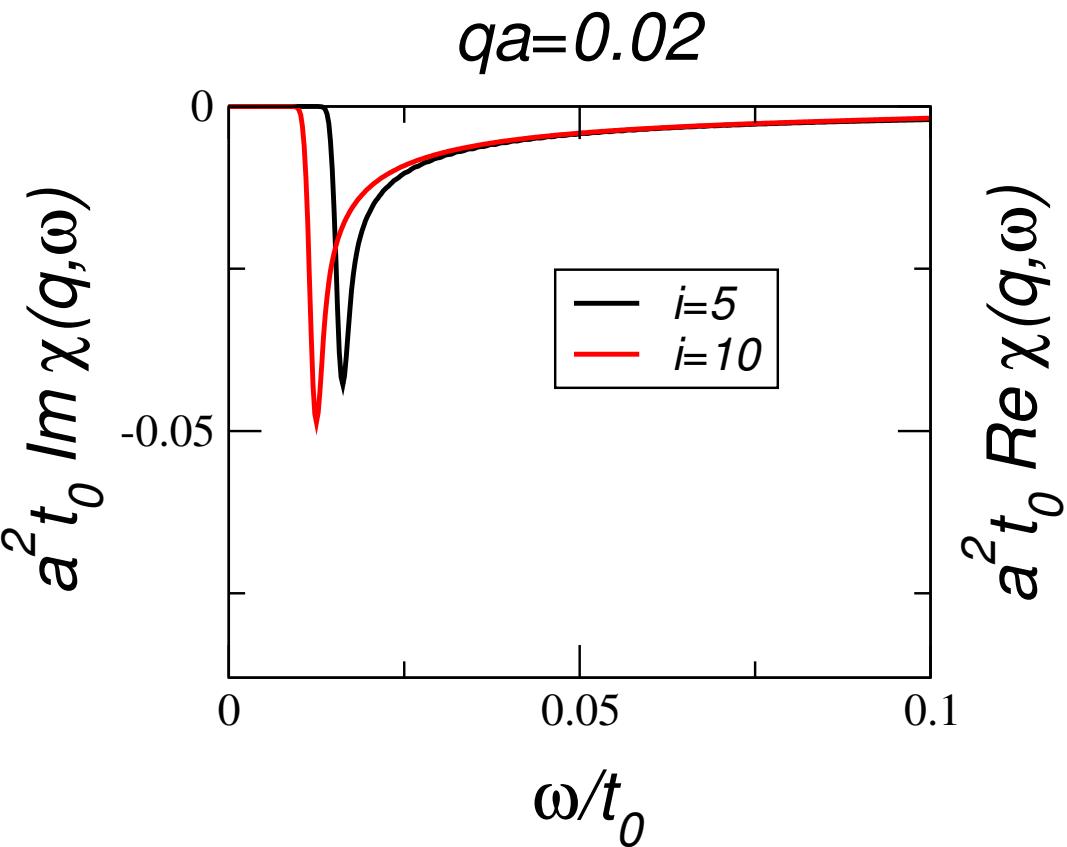
$$f_{\mathbf{G}, \mathbf{G}'}^{n,m;\kappa}(\mathbf{k}, \mathbf{q}) = \langle \mathbf{k}, n; \kappa | e^{-i(\mathbf{q}+\mathbf{G})\cdot\hat{\mathbf{r}}} | \mathbf{k} + \mathbf{q}, m; \kappa \rangle \langle \mathbf{k} + \mathbf{q}, m; \kappa | e^{i(\mathbf{q}+\mathbf{G}')\cdot\hat{\mathbf{r}}} | \mathbf{k}, n; \kappa \rangle$$

## Static response of twisted bilayer graphene

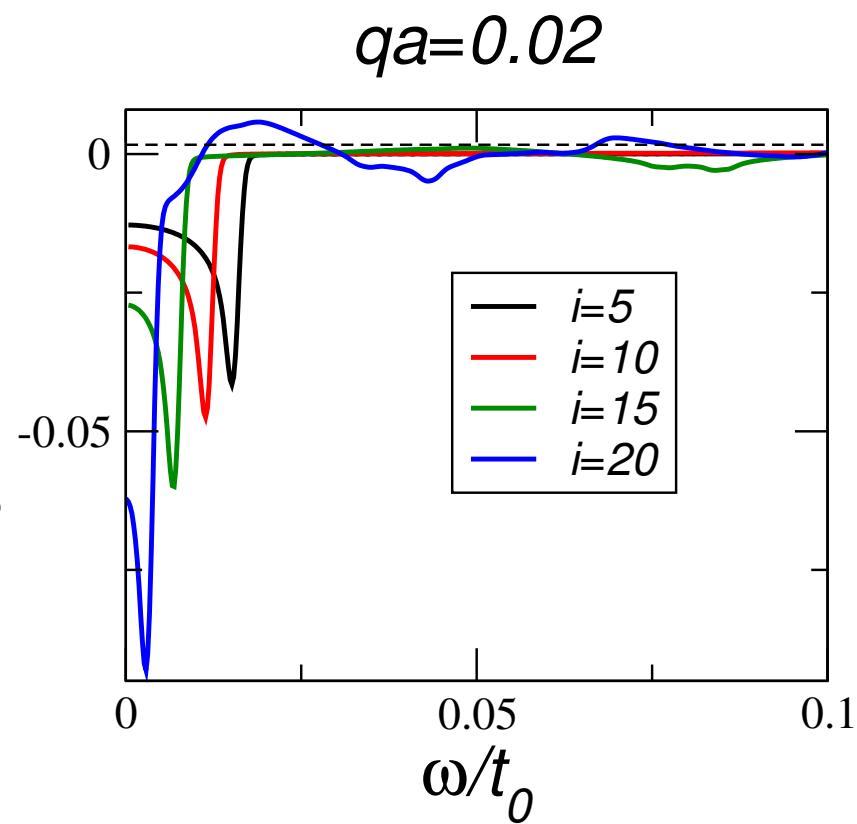
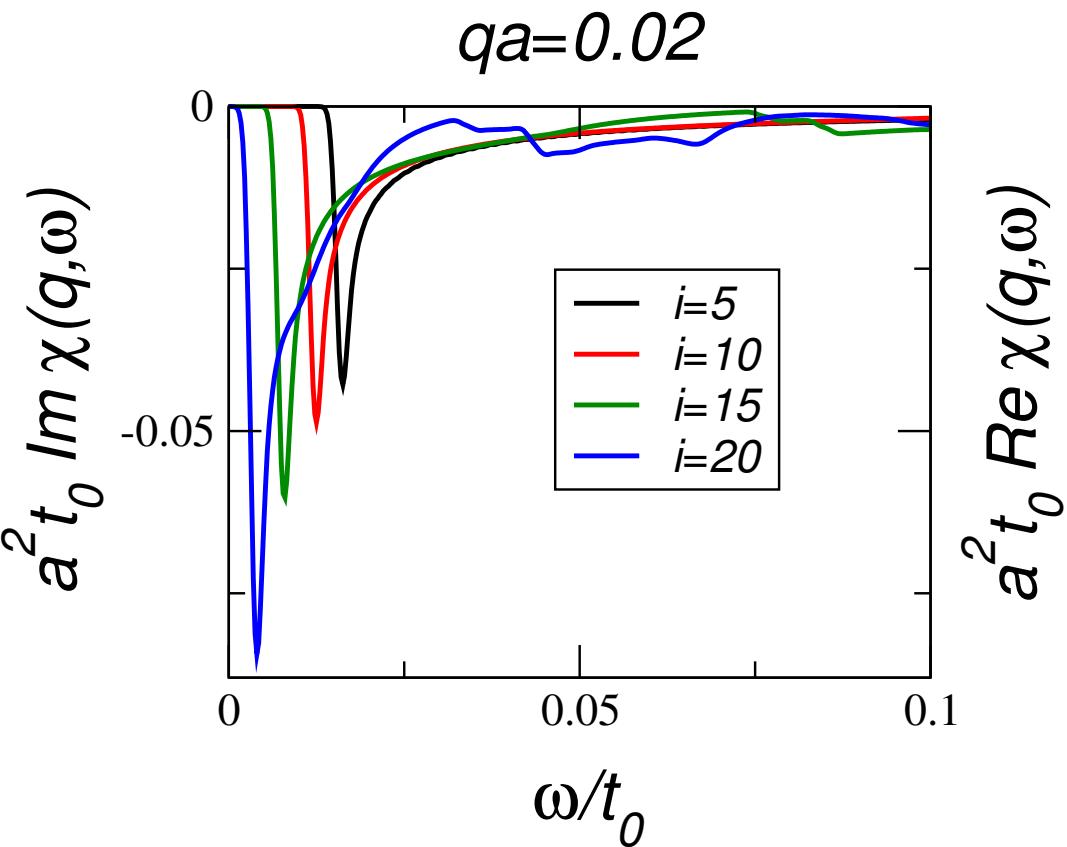
## Crossover from linear to quadratic behavior



## Dynamical response of twisted bilayer graphene

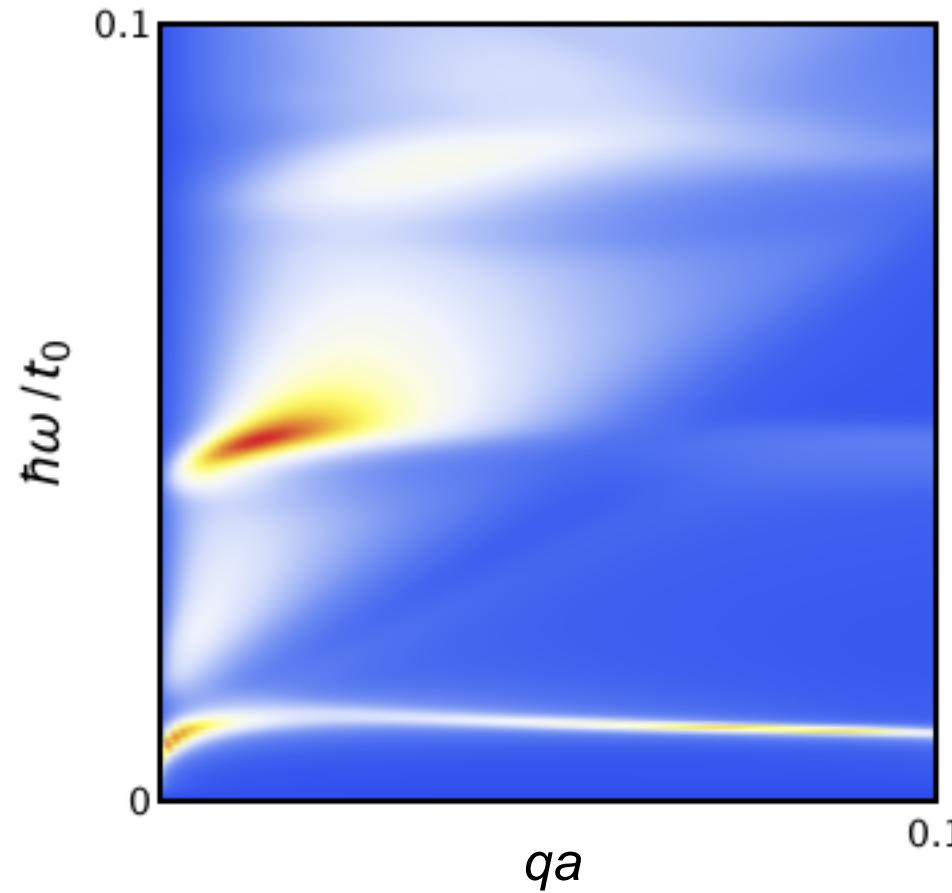
Crossover at  $i=15-20$ :

## Dynamical response of twisted bilayer graphene

Crossover at  $i=15-20$ :

## Loss function for i=25

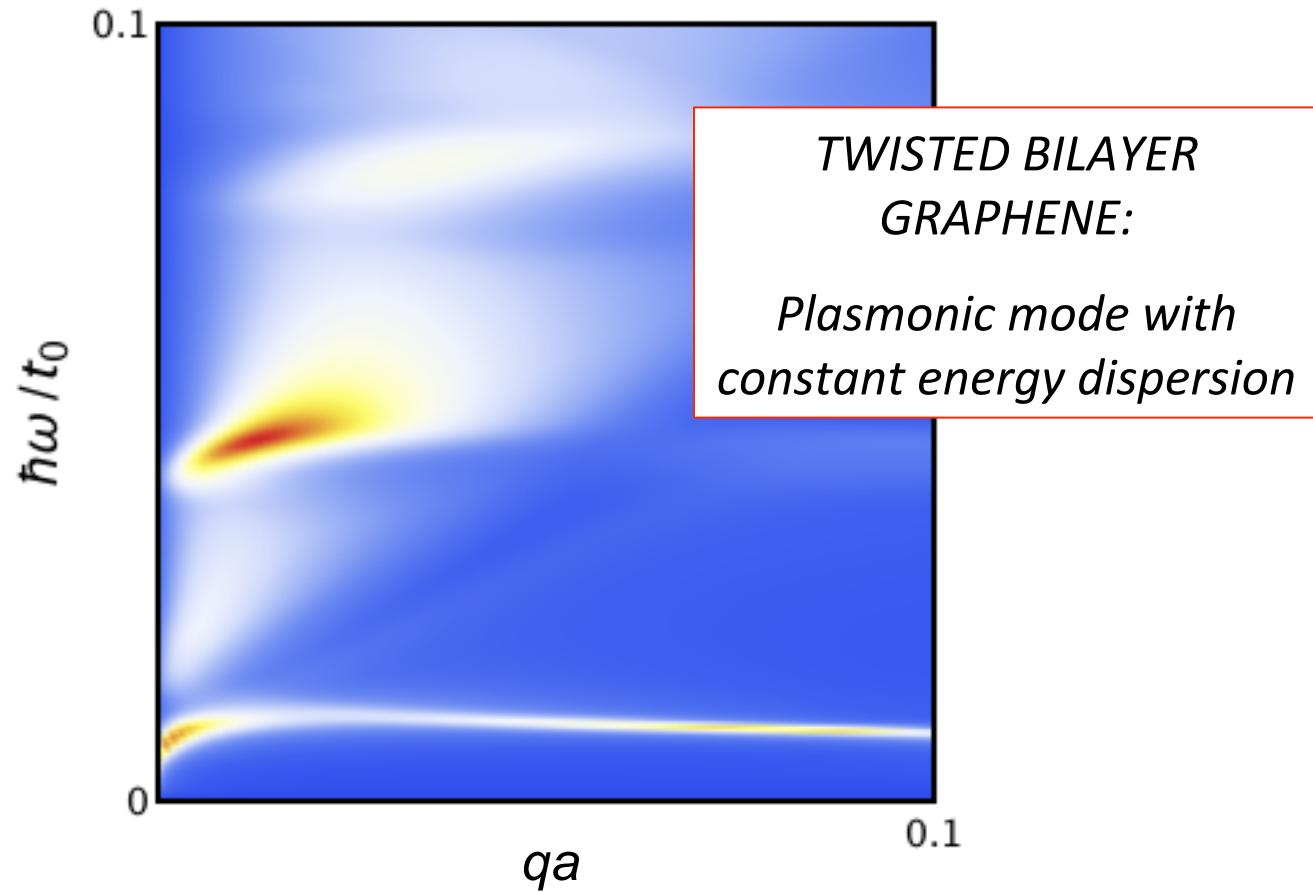
Quasi-flat plasmonic bands for  $\theta \approx 1.6^\circ$ :



T. Stauber and H. Kohler, Nano Lett. **16**, 6844 (2016)

## Loss function for i=25

Quasi-flat plasmonic bands for  $\theta \approx 1.6^\circ$ :



T. Stauber and H. Kohler, Nano Lett. **16**, 6844 (2016)

## Conclusions

1. For small enough twist angle, we find a novel plasmonic resonance of almost constant energy at zero doping. This mode can be tuned and quenched/enhanced by changing the twist angle and chemical potential, respectively.
2. The novel mode can be characterised as collective excitonic in-phase oscillations in a periodic, but quasi-confining potential surrounding the AA-stacked regions.
3. Twisted bilayer graphene resembles a new metamaterial with extraordinary properties in the THz to mid-infrared region leading to enhanced absorption and exponential amplification at constant energy reminiscent to Pendry's perfect lens, but without the need of left-handed materials.

Thank you for your attention!