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Anomalous Hooke's law in disordered graphene

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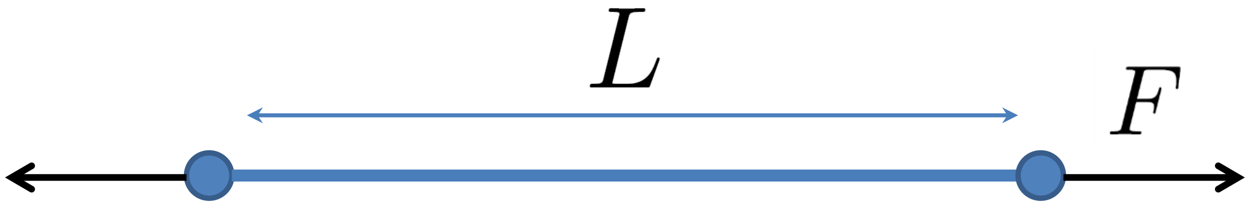
Gornyi, Kachorovskii, Mirlin, 2D Materials 4, 011003 (2017)

7th edition of the largest European Conference & Exhibition in Graphene and 2D Materials

Graphene 2017
March 28-31
Barcelona (Spain)

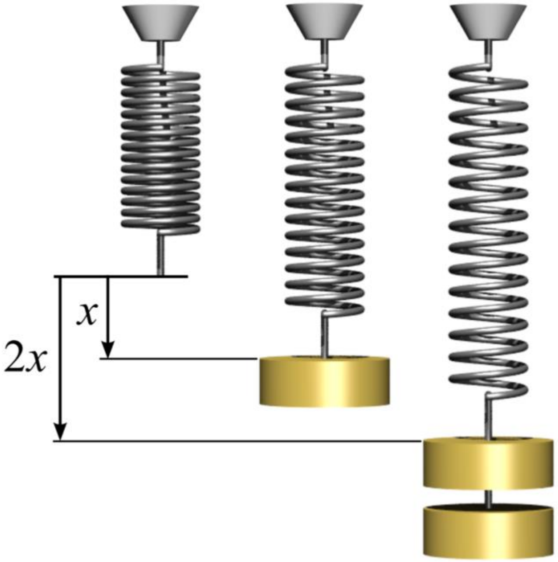
Hooke's law



$$\Delta L \propto F^{\alpha}$$

Hooke's law (1678): $\alpha = 1$ →

ut tensio, sic vis



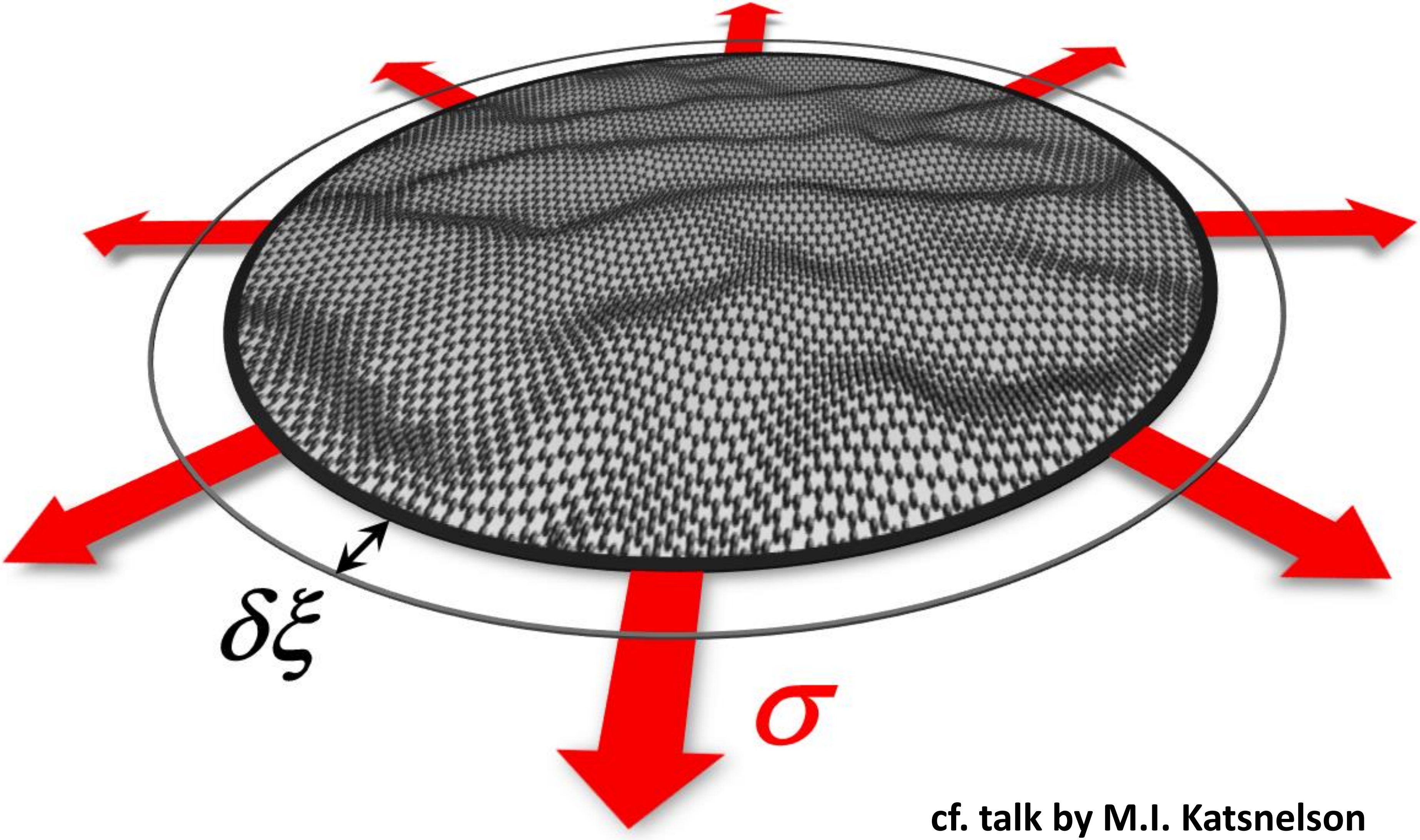
Graphene →

1) $\alpha \neq 1$ **anomalous Hooke's law**

2) $\alpha_{\text{clean}} \neq \alpha_{\text{disordered}}$

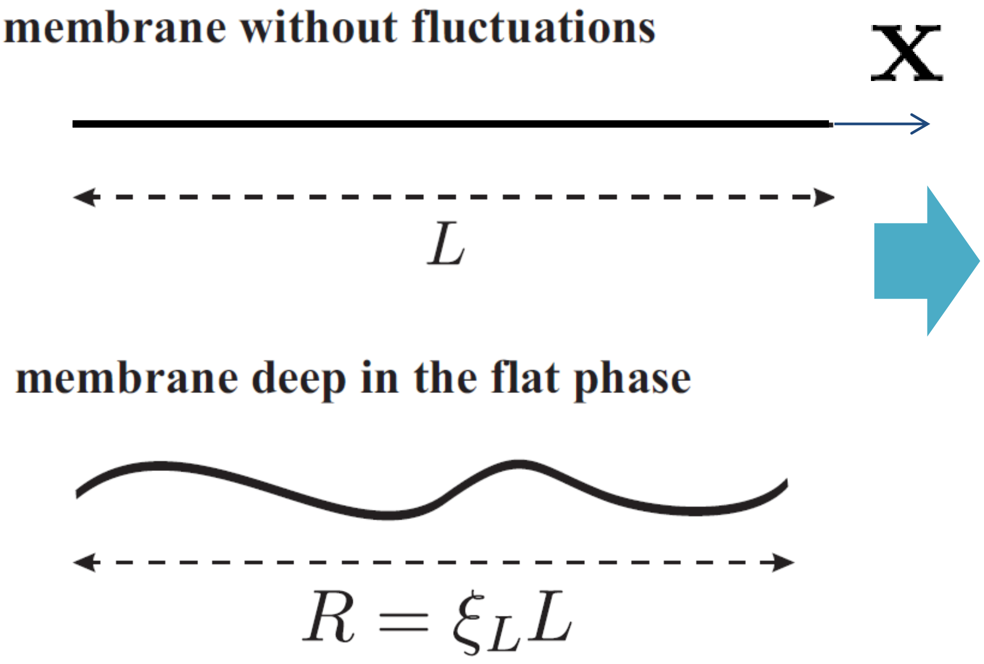
Suspended graphene:

Stretching vs. strain

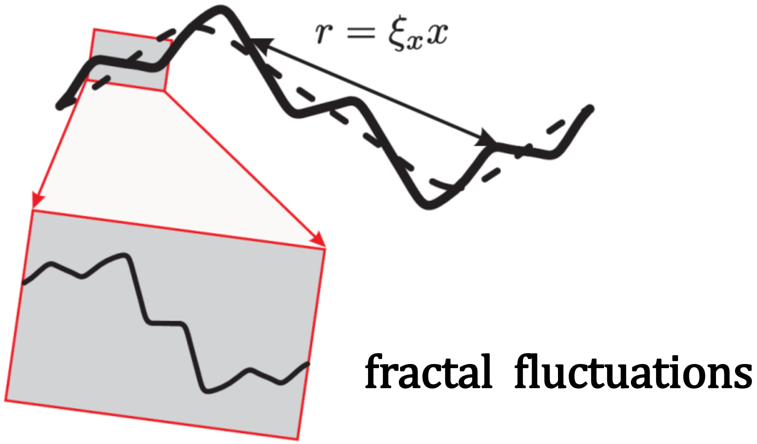


cf. talk by M.I. Katsnelson

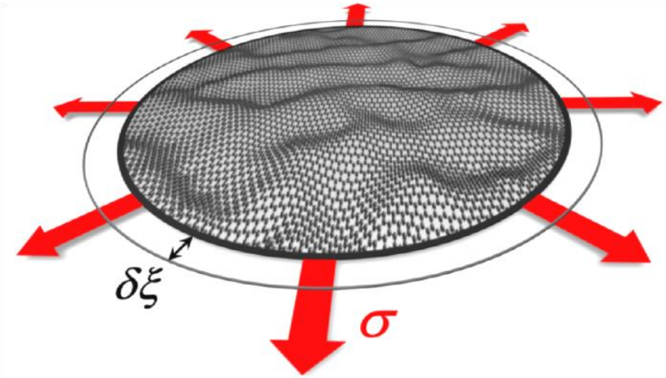
Global shrinking: “hidden area”



Membrane effect: **thermal fluctuations in y direction**
➔ shrinking in x direction
I.M. Lifshitz, JETP (1952)



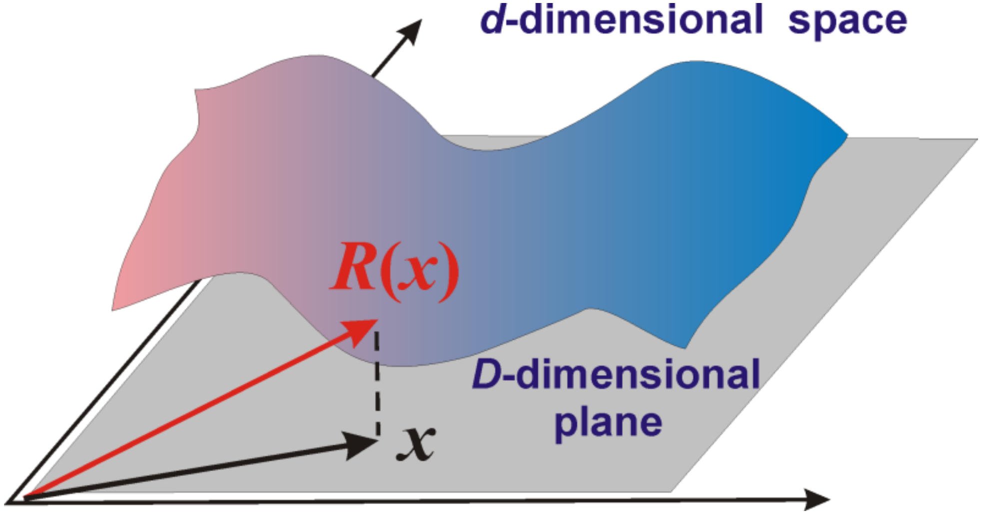
External tension σ “irons” thermal or static fluctuations:



Graphene as elastic membrane

$$\mathbf{R} = \underbrace{\xi\mathbf{x}}_{\text{global deformation}} + \underbrace{\mathbf{u} + \mathbf{h}}_{\text{in-plane and out-of-plane fluctuations}}$$

stretching parameter : $\xi < 1$



physical membranes: $d=3$, $D=2$

Elastic energy:

$$E = \frac{1}{2}\int d\mathbf{r}\left[\rho(\dot{\mathbf{u}}^2 + \dot{h}^2) + \varkappa(\Delta h)^2 + 2\mu u_{ij}^2 + \lambda u_{kk}^2\right]$$

strong anharmonicity

strain tensor:

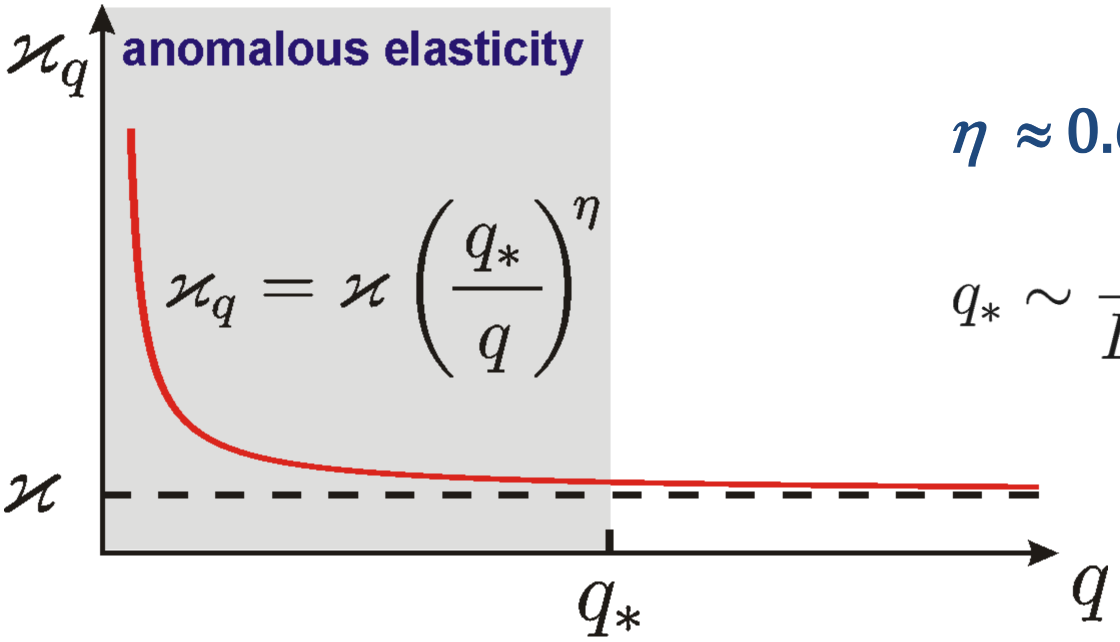
$$u_{ij} = \frac{1}{2}[\partial_i u_j + \partial_j u_i + (\partial_i h)(\partial_j h)]$$

Renormalization of bending rigidity

bending rigidity increases with increasing system size:

$$\frac{d\varkappa}{d\Lambda} = \eta\varkappa$$

David & Guitter, Europhys. Lett. (1988);
Aronovitz & Lubensky, PRL (1988);
Le Doussal & Radzihovsky, PRL (1992)



$\eta \approx$ 0.6 - 0.85

$$q_* \sim \frac{1}{L_*} \sim \sqrt{\frac{\mu T}{\varkappa^2}}$$

graphene: $\varkappa / T \approx 30$ **at** $T = 300$ K **,** $L_* \sim$ **5 ÷ 10 nm**

Clean membrane: Anomalous Hooke's law

Equation of state: $$\frac{\sigma}{\mu+\lambda} = \xi^2 - 1 + \frac{d_c T}{4\pi}\int_0^{q_{\rm uv}} \frac{q\mathrm{d}q}{\varkappa_q q^2 + \sigma}$$

$$\xi^2 - \xi_T^2 \propto \sigma^\alpha$$

$$\alpha = \frac{\eta}{2-\eta}$$

anomalous Hooke's law at SMALL (!!!) tension:

$$\sigma < \sigma_* \sim \mu \frac{T}{\varkappa}$$

Guitter, David, Leibler, Peliti, PRL (1988);
Aronovitz, Colubovic, Lubensky J. Phys. France (1989);
Gornyi, Kachorovskii, Mirlin, 2D Materials (2017)

Disordered graphene: Random curvature

Gornyi, Kachorovskii, Mirlin, PRB 92, 155428 (2015)

$$E = \int d^2\mathbf{x} \left\{ \frac{\varkappa}{2} [\Delta \mathbf{h} + \boldsymbol{\beta}(\boldsymbol{x})]^2 + \mu u_{ij}^2 + \frac{\lambda}{2} u_{ii}^2 \right\}$$

↑ random curvature (most relevant disorder)

$$P(\boldsymbol{\beta}) = Z_{\boldsymbol{\beta}}^{-1} \exp\left(-\frac{1}{2b} \int \beta^2(\mathbf{x}) d^2\mathbf{x} \right)$$

b – strength of disorder

from flexural phonons to static out-of-plane fluctuations (**ripples**):

$$\frac{T}{\varkappa} \to b, \quad \eta \to \frac{\eta}{4}$$

Scaling in disordered graphene

$$\frac{d\xi^2}{d\Lambda} = -\frac{1}{4\pi}\left(\frac{T}{\varkappa} + b\right)$$

➡

$$f = \frac{b\varkappa}{T}$$

f ≫1 → ripples dominate

f ≪ 1 → thermal fluctuations (flexural phonons) dominate

strongly disordered membrane

f ≫1

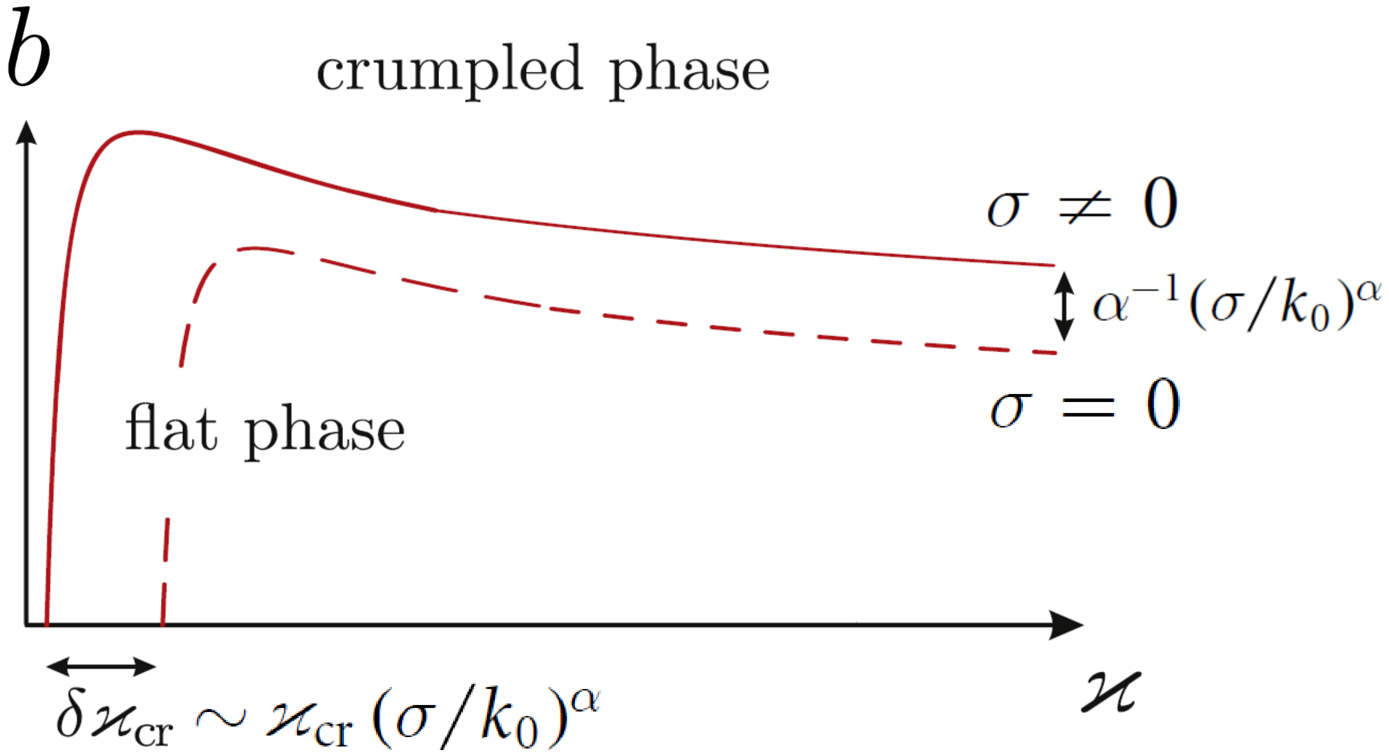
$$\frac{d\varkappa}{d\Lambda} = \eta\varkappa\frac{1+3f+f^2}{(1+2f)^2}$$

➡

$$\frac{d\varkappa}{d\Lambda} = \frac{\eta}{4}\varkappa$$

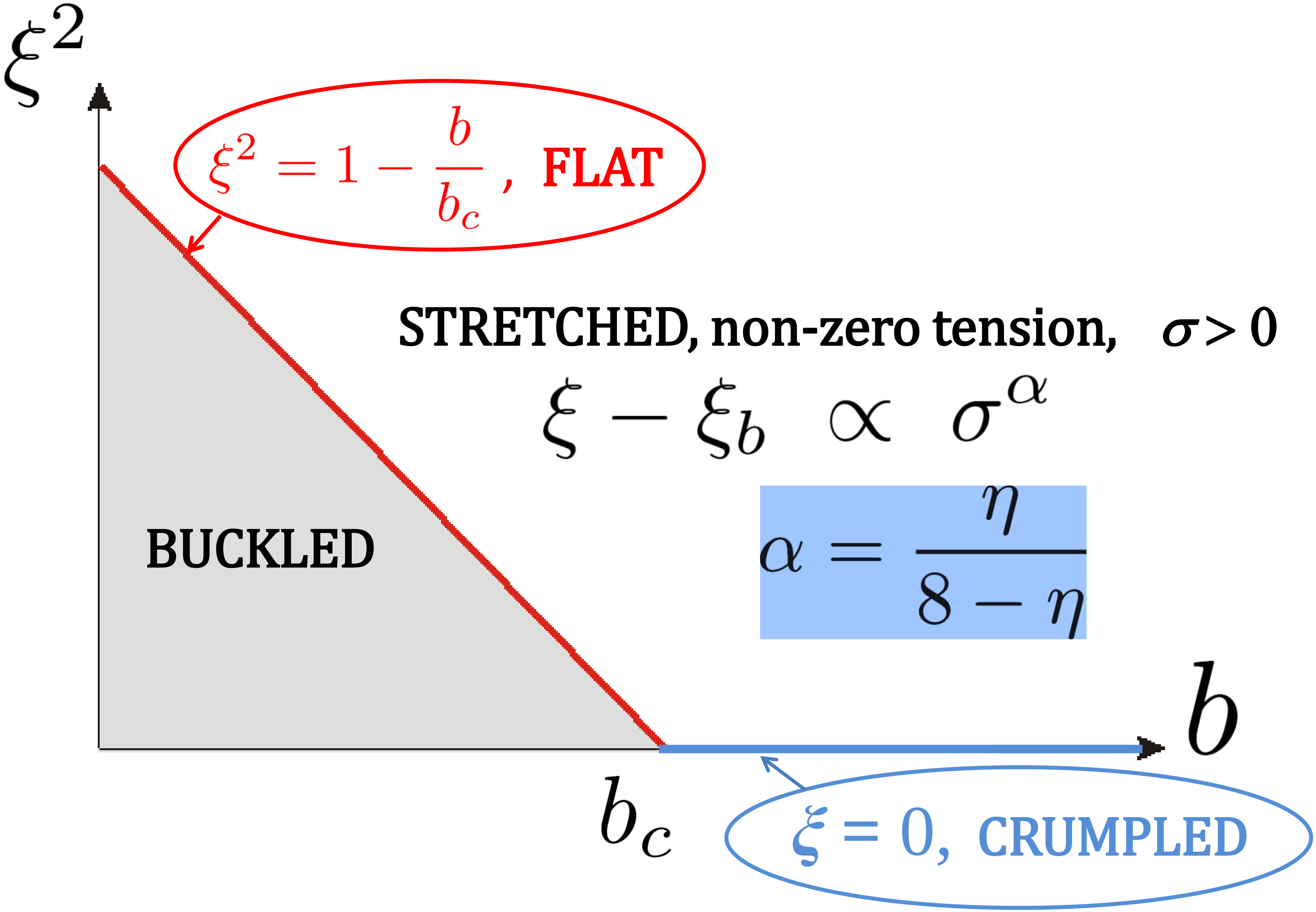
$$df/d\Lambda = -3\eta/4$$

Disordered graphene: Phase diagram



bare stiffness of graphene: $k_0 = 2(\mu + \lambda) \approx 400\ \mathrm{N\ m}^{-1}$

Crumpling and buckling in disordered graphene



ξ^2

$\xi^2 = 1 - \frac{b}{b_c}$, **FLAT**

STRETCHED, non-zero tension, $\sigma > 0$

$$\xi - \xi_b \propto \sigma^{\alpha}$$

$$\alpha = \frac{\eta}{8 - \eta}$$

BUCKLED

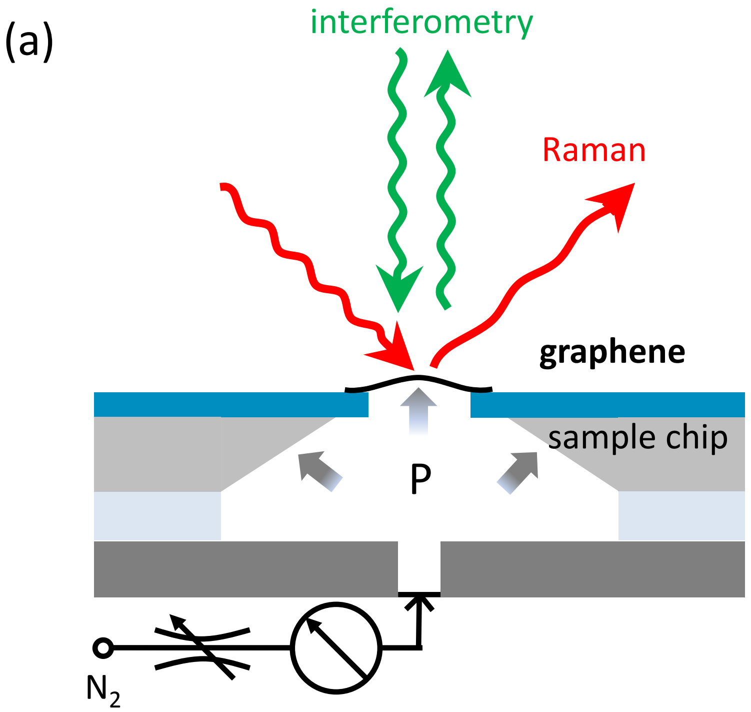
b

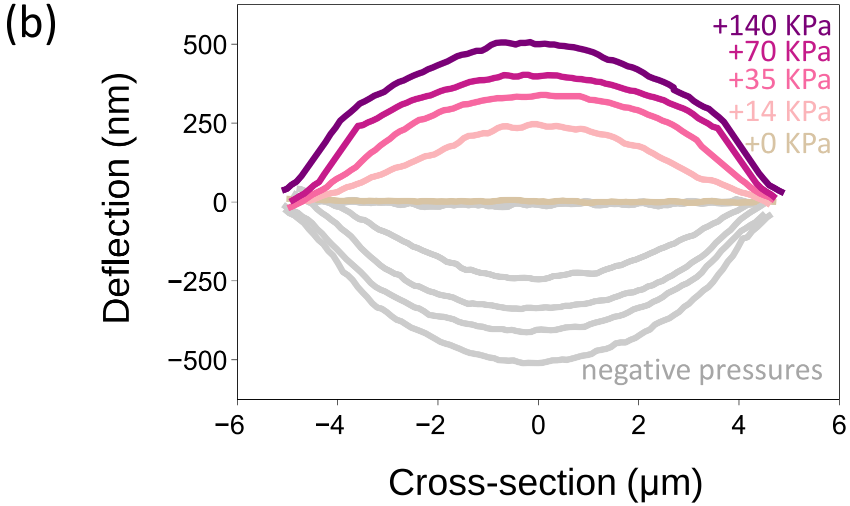
b_c

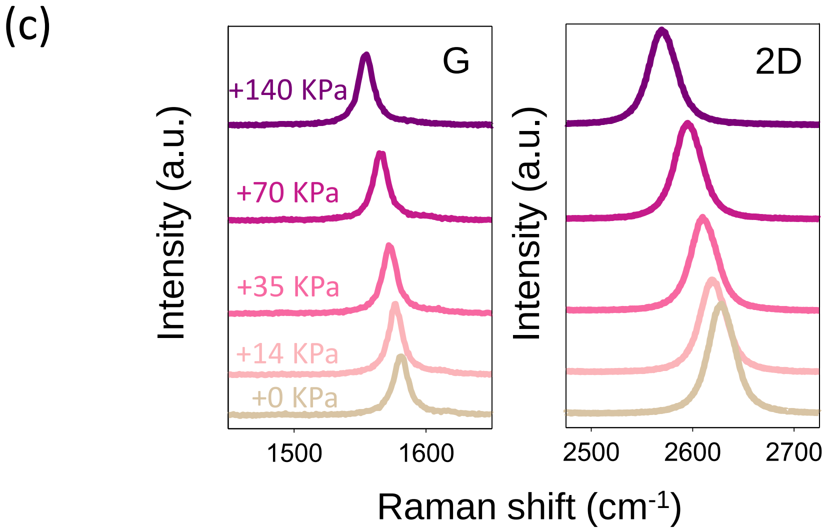
$\xi = 0$, **CRUMPLED**

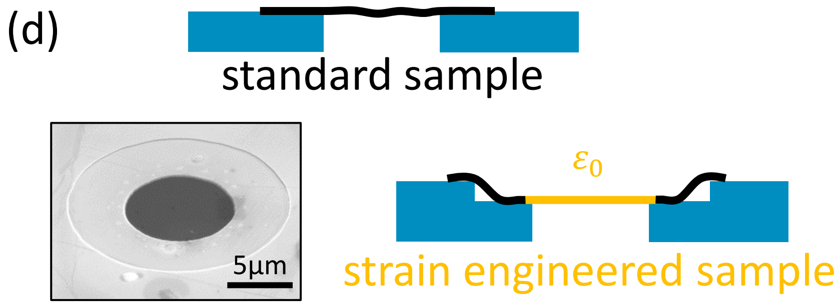
Anomalous Hooke's law: experiment

Nicholl, Conley, Lavrik, Vlassiouk, Puzyrev, Sreenivas, Pantelides, Bolotin, Nature Comm. (2015)



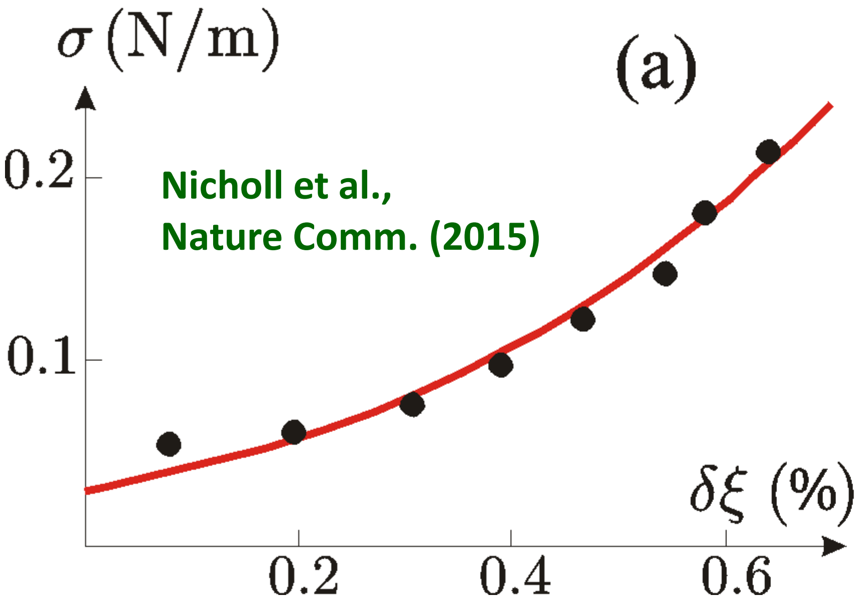






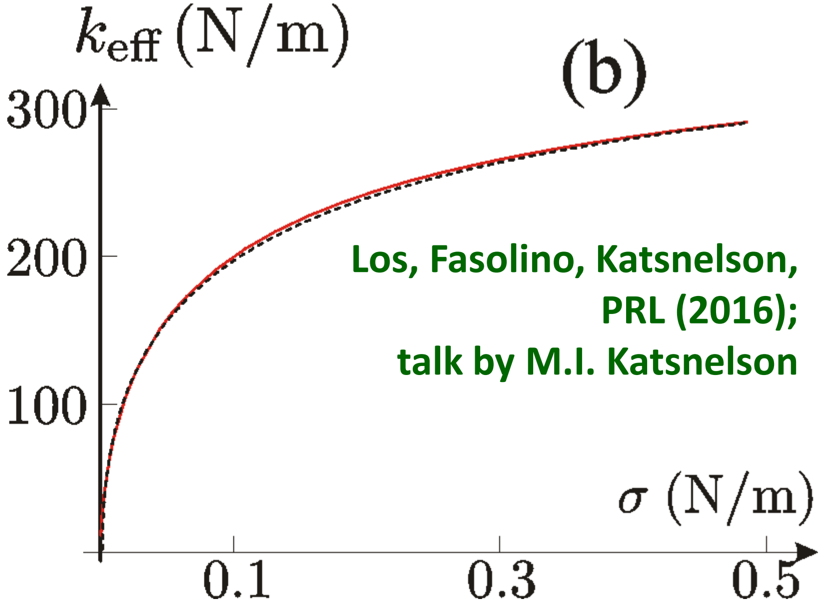
Comparison with experiment and simulations

Analytical theory (red curve): Gornyi, Kachorovskii, Mirlin, 2D Materials (2017)



Nicholl et al., Nature Comm. (2015)

disordered graphene



Los, Fasolino, Katsnelson, PRL (2016); talk by M.I. Katsnelson

clean graphene

$$k_{\text{eff}} = \partial\sigma/\partial\xi \simeq k_0 \frac{(\sigma/\sigma_*)^{1-\alpha}}{1+(\sigma/\sigma_*)^{1-\alpha}} \neq \text{const}$$

Summary

- **Analytical theory** of elasticity in disordered graphene
- **Anomalous Hooke’s law:** stretching of a graphene flake is a nonlinear function of the applied tension
- **Disorder** does not destroy anomalous Hooke’s law, but changes its critical exponent
- **Scaling of the stiffness** obeys a fractal power law and is governed by static ripples at low T
- **Agreement** with experiment and simulations
- **Outlook:** Poisson’s ratio in disordered graphene